Proceedings of the

10th International Symposium on Vibrations of Continuous Systems

Stanley Hotel, Estes Park, Colorado, USA

26-31 July 2015

Preface

The International Symposium on Vibrations of Continuous Systems (ISVCS) is a forum for leading researchers from across the globe to meet with their colleagues and to present both old and new ideas in the field. Each participant has been encouraged either to present results of recent research or to reflect on some aspect of the vibration of continuous systems which is particularly interesting, unexpected or unusual. This type of presentation is meant to encourage participants to draw on understanding obtained through many years of research in the field.

The 10th ISVCS takes place on 26-31 July 2015 at the Stanley Hotel, Estes Park, Colorado USA, which was the venue for the 1st ISVCS in 1997. It focuses on the vibrations of the vibrations of the fundamental structural elements: strings, rods, beams, membranes, plates, shells, bodies of revolution and other solid bodies of simple geometry. Structures composed of assemblies of structural elements are also of interest, especially if such structures display interesting or unusual response.

Typical days at the Symposium will consist of morning hikes or excursions in the local area, afternoon technical presentations and, in the evening, further technical discussions and social gatherings. The various outings and social gatherings provide important opportunities for relaxed and informal discussion of technical and not-so-technical topics surrounded by the natural beauty of the Rocky Mountain National Park along the Big Thompson River.

This volume of Proceedings contains 20 short summaries of the technical presentations to be made at the Symposium, as well as short biographical sketches of the participants.

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The 9th International Symposium:
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Details of the Proceedings of the past Symposia can be found at http://www.isvcs.org

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Biosketches

The Dynamic Stiffness Method: A Review of the State of the Art

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Abstract

For over half a century, the static and dynamic analyses of structures have continued to be routinely carried out by the finite element method (FEM) which is without doubt a universal tool in structural mechanics. Although the FEM originated as a break-through in solid mechanics in the late fifties and early sixties, it has subsequently infiltrated in other disciplines such as fluid mechanics, material science and electrical engineering with considerable success. In structural engineering applications, the choice of shape functions which characterise the deformed shape of a typical finite element is crucial and a fundamental consideration in FEM modelling. For free vibration analysis of structures, the stiffness and mass properties of individual elements derived from the assumed shape functions are assembled in FEM to form the overall stiffness (**[K]**) and mass (**[M]**) matrices of the final structure and then the traditional linear eigenvalue problem e.g., $|\mathbf{[K]} - \lambda \mathbf{[M]}| = 0$ is formulated where the square root of λ gives the natural frequencies of the structure. Following this eigen-solution procedure, the mode shapes are recovered in the usual way. Although the FEM is an approximate method based on chosen shape functions, the results are expected to be sufficiently accurate with increasing number of elements. The FEM is numerically intensive and the degrees of freedom typified by the order of **[K]** and **[M]** matrices dictate the number of eigenvalues that can be computed. The accuracy of results will obviously depend on the quality of the elements. The higher order natural frequencies from any FEM analysis will understandably be less accurate. There are numerous texts on FEM and Zienkiewicz [1] is one of the pioneers in this respect. Structural analysis is dominated by the FEM and against this continuing dominance, there appears to be an elegant and powerful alternative to FEM, particularly when solving free vibration problems of structures. This alternative is the so-called dynamic stiffness method (DSM) which has much better modelling capability than the FEM. Unlike the FEM, the DSM relies on frequency dependent exact shape functions of structural elements derived from their governing differential equations of motion in free vibration. Due to the exactness of the shape functions, the results obtained from the DSM are often called exact. In the DSM, separate mass and stiffness matrices are not derived, but a single frequency dependent element stiffness matrix which contains both the mass and stiffness properties of the element is exploited as the main building block to obtain the overall dynamic stiffness matrix of the final structure. The assembly procedure in the DSM is essentially the same as the FEM, but the formulation leads to a nonlinear eigenvalue problem. The results obtained from the DSM are independent of the number of elements used in the analysis. For instance, one single structural element can be used in the DSM to compute any number of its natural frequencies and mode shapes to any desired accuracy which is of course, impossible in the FEM or in any other approximate methods. It is considered by many researchers that the DSM is the ultimate bench mark in free vibration analysis of structures.

The DSM was pioneered by Kolousek in the early forties [2, 3] who provided the elements of the dynamic stiffness matrix of a Bernoulli-Euler beam for the first time. At this stage, there was no robust solution technique for the DSM which generally leads to a transcendental eigenvalue problem and researchers often had to rely on the cumbersome and tedious determinant plots involving highly irregular functions, causing formidable problems. Following Kolousek's work

[2-3], Wittrick and Williams [4] developed a powerful algorithm in the early seventies which effectively secured the foundation of the DSM [5] and it made a huge impact on the DSM researchers. The Wittrick-Williams algorithm is now an established solution technique in DSM. Akesson [6] and Williams and Howson [7] published computer programs in Fortran using the DSM and the Wittrick-Williams algorithm for free vibration analysis of plane frame structures. Subsequently, the dynamic stiffness matrices of Timoshenko beam [8] and axially loaded Timoshenko beam [9] were developed. The advances made in the dynamic stiffness formulation of beams led to the development of the computer program BUNVIS-RG [10] to analyse space frames. During a sustained period of developments, the DSM has been successfully applied to investigate complex structural systems which include beams [11-21], plates [22-24] and shells [25]. The DSM based computer program VICONOPT [26] with its optimising features can deal with the free vibration and buckling problems of isotropic and anisotropic plate assemblies. Nevertheless, the DSM literature is not as broad or diverse as the FEM, but the DSM has without doubt, matured sufficiently since its foundation to justify writing a review paper on the state of the art. The purpose of this paper is to undertake this task and give an overview of the historical developments of the DSM and highlight its future prospects and challenges ahead.

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The role of shear deformation in the tensile instability of beam-columns

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Abstract

Instability caused by tensile axial loads acting on beams, has been highlighted in a very few papers in the specific literature, precisely in the case of rubber bearing isolators [4], of highly shear deformable beams [6] and for beam-columns in presence of a single structural junction allowing transversal deflection discontinuities [5] or due to effects of the constraint's curvature [1]. The authors provided contributions on the subject by assessing the influence of multiple cracks [2] and also of multiple internal sliders [3] occurring along shear deformable beams on the tensile buckling phenomenon without considering any dynamic effect.
The main concept to be brought to light after the latter mentioned studies is that instability caused by tensile loads acting on beam-like structures can be disclosed if some sort of shear deformability is accounted for in beams.
On this topic many aspects are to be clarified yet. In this work the influence of shear deformation singularities, under the form of transversal deflection discontinuities, on the dynamic instability of beams under tensile loads is explored.
Precisely, a contribution to the knowledge of dynamic instability due to tensile dead load of a beam-column in presence of an arbitrary number of internal sliders endowed with translational elastic springs is offered. First, closed form expressions of the eigen-modes and the characteristic equation under the action of axial forces, for different boundary conditions, are originally derived and presented for the first time.
Furthermore, besides the case of conservative tensile dead load, a non conservative axial load of the Beck type has been also considered and the following relevant question is asked: Can a beam-column undergo tensile flutter instability ?
While tensile axial loads with conservative character induce divergence type instability, this paper shows that a cantilever beam-column, under a non conservative tensile load only, undergoes flutter instability, as well as under compressive loads, when transversal displacement discontinuities along the span are allowed. A comprehensive parametric analysis is conducted.
Finally, the contemporary presence of conservative and non conservative loads is investigated to study the reciprocal influence. Ranges where the structure switches from divergence to flutter instability, both in tension and in compression, are predicted and discussed in detail.
The dynamic behaviour of an Euler-Bernoulli beam subjected to an axial, either compressive ($N>0$) or tensile ($N<0$) dead load, as in Figure 1, characterised by the presence of multiple internal sliders at $\xi_i, i=1,n$, endowed with elastic springs, can formulated by means of the following governing equation with respect to the normalised abscissa $0\le\xi\le1$:

$$\phi^{IV}(\xi)+\sigma^2\phi^{//}(\xi)-\mu^4\phi(\xi)=-\sum_{i=1}^{n}\lambda_i\phi^{///}(\xi_i)\delta^{///}(\xi-\xi_i) \tag{1}$$

where ϕ is the transversal displacement mode shape, σ^2, μ^4 are the axial load and frequency parameters, respectively, λ_i is the normalised translational spring compliance, $\delta(\xi - \xi_i)$ is the Dirac's delta distribution and the apex indicates the derivative with respect to ξ.

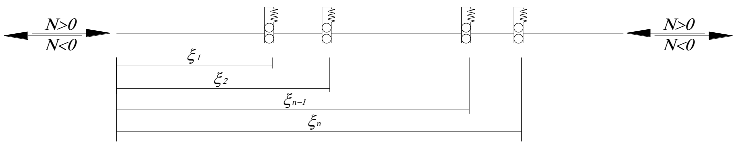

Figure 1. A beam with multiple internal elastic sliders.

The closed form solution of Equation (1) is derived in this work by making use of the theory of distributions, but not reported here for brevity. The great advantage of the proposed closed form solution is that, regardless of the number of transversal displacent discontinuities, a costless extensive parametric analysis can be conducted without enforcement of any continuity condition. Furthermore, the obtained explicit closed form solution of the problem at hand is crucial for the formulation of the dynamic stiffness matrix of the beam in Figure 1 to analyse frame structures with multiple shear singularities.
In Figure 2a the axial load-frequency parameter interaction diagram is depicted for a clamped-clamped beam in presence of two internal sliders. In the latter figure a switch from a symmetric $\mathcal{A}$ to an anti-symmetric $\mathcal{B}$ mode under a tensile load, that is not encountered under compressive load, is shown. Furthermore, if an increasing number of sliders is accounted for in the solution, is has to be highlighted that the axial load-frequency parameter interaction diagram reproduces the case of Timoshenko beam as shown in Figure 2b.

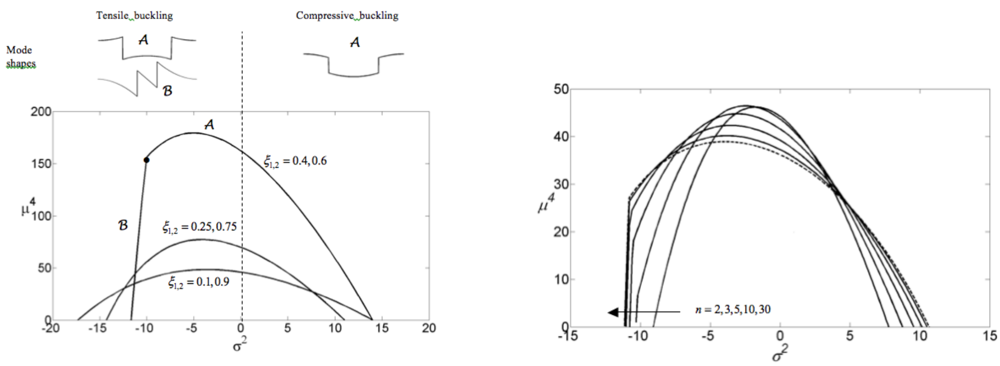

Figure 2. A clamped-clamped Euler-Bernoulli beam with internal elastic sliders: a) two sliders; b) an increasing number *n* of sliders compared with a Timoshenko beam model (dashed line).

Besides providing the divergence critical tensile load together with the relevant instability mode shapes, the comparison reported in Figure 2b demostrates that the discountinuos Euler-Bernoulli beam-column can be considered as the discrete counterpart of the Timoshenko model. In other words the elastic internal sliders, which allow transversal deflection discontinuities, can be interpreted as concentrated shear deformations occurring along the span and can be addressed to as shear deformation singularities.
Furthermore, the closed form solution of Equation (1) can also account for the presence of non conservative, follower type, tensile axial load as in the cantilever beam-column in Figure 3a.
It is well known that the instability phenomenon of the flutter type characterised by oscillations of increasing amplitude can be reached in case of compressive load. Here, the question whether

the same type of instability can be caused by the application of follower tensile loads is addressed and answered affirmatively. Precisely, a coalescence of the first and second frequency in the axial load-frequency parameter interaction diagrams is recognised for the beam-column in Figure 3a as obtained and shown in Figure 3b for an increasing number of internal elastic sliders.

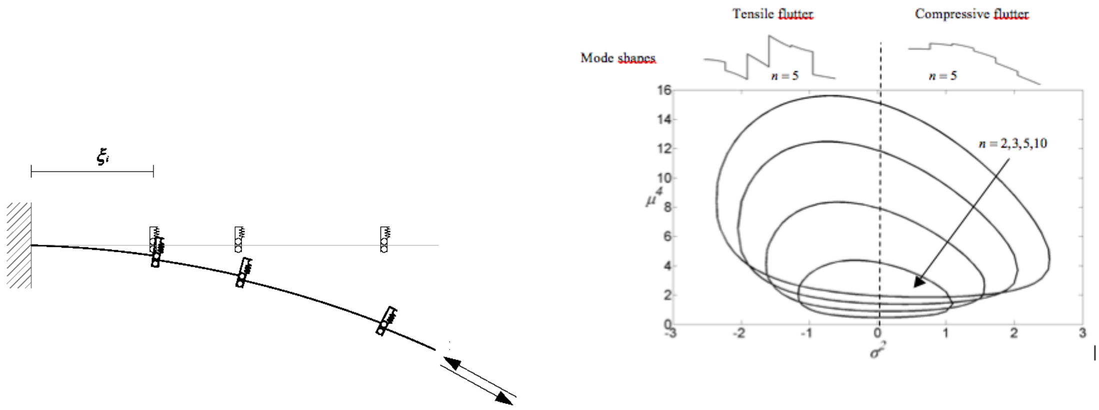

Figure 3. a) A cantilever beam-column with multiple internal elastic sliders subjected to a follower force; b) Axial load-frequency parameter diagram for an increasing number of elastic sliders.

Finally, a thourough analysis of the beam-column in Figure 3a is pursued by considering, together with the follower type load, the contemporary presence of a conservative load. In particular, the reciprocal influence is assessed by introducing the so called non conservative parameter defined as the ratio of the angle of the total external force and the inclination angle of the tangent to the deformed beam axis. Ranges where the structure switches from divergence to flutter instability, both in tension and in compression, can be predicted and discussed in detail.

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Vibration analysis of metallic and composite damaged structures by Component-Wise models

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Abstract

This work proposes an innovative approach that is based on higher-order beam models for the damage analysis of metallic and composite structures. The present 1D formulation stems from the Carrera Unified Formulation (CUF) and it leads to a Component-Wise (CW) modelling.

According to CUF, the accuracy of the analysis is a parameter of the formulation. The displacement field is, in fact, expressed as an arbitrary expansion of the generalized unknowns via user-defined cross-sectional functions, F_τ. Formally,

$$\mathbf{u}(x,y,z) = F_\tau(x,z)\mathbf{u}_\tau(y), \qquad \tau = 1,2,....,M \tag{1}$$

where $\mathbf{u}_\tau$ is the vector of the *generalized displacements* and M stands for the number of terms used in the expansion. Taylor-like polynomials, i.e. power series of the coordinates x and z of generic order N, can be used as basis functions (F_τ) to enrich the beam kinematics. Taylor Expansion (TE) models are not described in this paper but they can be found in [1]. Here, the attention is focused on CW CUF models, which make use of Lagrange polynomials as F_τ expanding functions. Lagrange polynomials are reported in many reference books, see for example [2]. In this paper, nine-point cubic (L9) Lagrange polynomials are employed. Nevertheless, thanks to the hierarchical capability of CUF, the order and the number of the Lagrange elements used to discretize the beam cross-section can be arbitrarily varied without changing the formal expression of the problem. In the simple case of single-L9 beam model, the displacement field is

$$\begin{aligned} u_x &= F_1\, u_{x_1} + F_2\, u_{x_2} + F_3\, u_{x_3} + F_4\, u_{x_4} + F_5\, u_{x_5} + F_6\, u_{x_6} + F_7\, u_{x_7} + F_8\, u_{x_8} + F_9\, u_{x_9} \\ u_y &= F_1\, u_{y_1} + F_2\, u_{y_2} + F_3\, u_{y_3} + F_4\, u_{y_4} + F_5\, u_{y_5} + F_6\, u_{y_6} + F_7\, u_{y_7} + F_8\, u_{y_8} + F_9\, u_{y_9} \\ u_z &= F_1\, u_{z_1} + F_2\, u_{z_2} + F_3\, u_{z_3} + F_4\, u_{z_4} + F_5\, u_{z_5} + F_6\, u_{z_6} + F_7\, u_{z_7} + F_8\, u_{z_8} + F_9\, u_{z_9} \end{aligned} \tag{2}$$

where $u_{x_1},...,u_{z_9}$ are the displacement variables of the problem and represent the translational displacement components of each of the nine points of the L9 element; $F_1,...,F_9$ are the Lagrange polynomials. Further details about CUF models based on Lagrange polynomials expansions can be found in [3].

In this work, a finite element approximation is adopted; the generalized displacements are thus expressed as a linear combinations of the nodal unknowns, $\mathbf{q}_{\tau i}$, through classical shape functions N_i. The governing equations are derived by means of the principle of virtual displacements. A compact form of the virtual variation of the strain energy can be obtained as shown in [1].

$$\delta L_{\text{int}} = \delta \mathbf{q}_{\tau i}^T \mathbf{K}^{ij\tau s} \mathbf{q}_{sj} \tag{3}$$

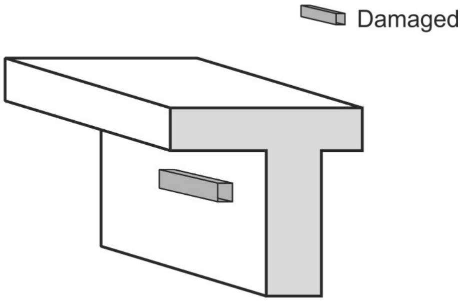

Figure 1: Locally damaged structure.

where $\mathbf{K}^{ij\tau s}$ is the stiffness matrix in the form of the fundamental nucleus. Superscripts indicate the four indexes exploited to assemble the matrix: i and j are related to the shape functions, τ and s are related to the theory expansion functions. The fundamental nucleus is a 3×3 array whose components can be found in [1, 3]. Matrix $\mathbf{K}^{ij\tau s}$ has to be expanded versus the four indexes to obtain any desired class of refined beam finite elements. Similarly, the fundamental nucleus of the mass matrix, $\mathbf{M}^{ij\tau s}$, can be easily obtained from the virtual variation of the work of inertial loadings, see [4].

A basic damage modelling approach is adopted in this work. Figure 1 shows an example of locally damaged structure. In the damaged zone, the material characteristics were modified according to the following formula:

$$E_d = d \times E \text{ with } 0 \leq d \leq 1 \tag{4}$$

i.e.;

$$E_0 = E; E_{0.9} = 0.9 \times E; ...; E_{0.1} = 0.1 \times E \tag{5}$$

A cantilever I shaped cross-section beam is discussed as a numerical example and it is shown in Figure 2. The main dimensions of the structure were: height, $h = 0.1$ m; width, $w = 0.1$ m; thickness of the flanges and the web, $t = 2$ mm; length, $L = 1$ m. The whole beam was made of an aluminium alloy ($E = 75$ GPa, $\nu = 0.33$, $\rho = 2700$ Kg m^{-3}). Damage was introduced in the whole top flange. Table 1 shows the first five natural frequencies of the structure subjected to different damage intensities. A CUF CW model built with 8L9 elements on the cross-section is compared to classical beam theories (Euler-Bernoulli, EBBM, and Timoshenko beam models, TBM) and to a 2D finite element plate model obtained with the commercial code Abaqus. Figure 3 shows the mode shapes for the un-damaged structure by the proposed CW CUF model. Also,

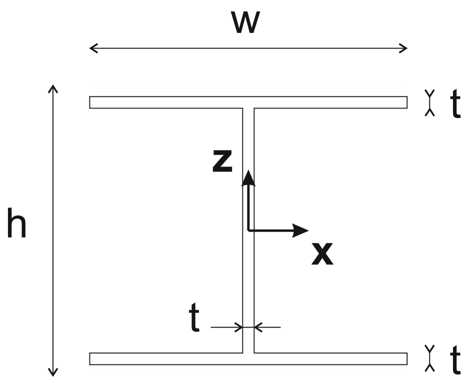

Figure 2: Cross-section of the I shaped beam.

Table 1: First natural frequencies (Hz) for different models and damage intensities, I-section beam.

Models	EBBM (193)*			TBM (305)			8 L9 CUF (4743)			2D Abaqus (27972)		
$(1-d)$	0	0.5	0.9	0	0.5	0.9	0	0.5	0.9	0	0.5	0.9
f_1	69.9	60.5	51.8	69.7	60.3	51.7	69.7	51.7	26.4	69.2	51.1	26.2
f_2	127.0	109.2	84.1	125.6	108.1	83.5	73.8	71.6	70.9	72.8	70.7	70.0
f_3	434.7	376.4	322.4	424.7	368.6	346.5	122.7	106.4	82.4	119.8	103.9	80.3
f_4	776.3	666.2	510.6	723.9	626.0	488.5	239.1	234.2	136.9	232.3	226.3	134.4
f_5	1202.2	1202.3	891.7	1142.0	993.8	855.8	252.8	243.3	199.7	246.3	236.0	183.4

*The number of degrees of freedom are given in brackets

the same figure graphically shows the effects of damages in the top flange on the first five natural frequencies. The analysis clearly demonstrates that refined beam models are mandatory to detect the damage effects. Moreover, it is clear that the results from 1D CUF models perfectly match those from a 2D FEM model with very low computational costs. The enhanced capabilities of refined CW models in dealing with damaged complex structures [5] and composite materials [6] will be further discussed during the 10th International Symposium on Vibrations of Continuous Systems.

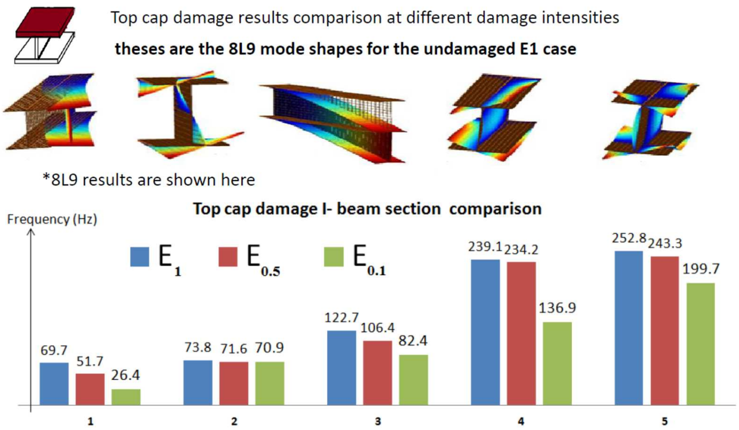

Figure 3: Mode shapes and damage effects on the natural frequencies by the 8 L9 CUF model

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Exact and distorted similitudes of vibroacoustic systems: where are we ?

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Abstract

The concept of distorted similitude can provide a powerful tool to predict the behavior of a given system by using an appropriate scaled model and the related scaling laws. These can be investigated, for linear systems, as a variation of the original sizes resulting in a modification of the distribution of the natural frequencies. Thus, the possibility to define exact and distorted similitudes can be pursued through theoretical and numerical models and further with dedicated experimental validations. The theoretical and numerical findings show that these distorted similitudes are feasible but the accuracy is not necessarily guaranteed when these laws are applied to real (experimental) structures.

In the last edition of ISCVS at Courmayeur, Italy, the main topics of the similitude procedure for vibroacoustic systems were presented and deeply discussed, [1]. The good feeling received there, together with the right questions still to be answered formed a strong motivation to continue in investigating this subtle but fascinating topic, [2]-[6] . This paper just presents a focus on what has been done and the main steps to be performed. At the moment, some investigations can be considered as concluded:

- the response of a generic acoustic volume, Fig. 1;
- the response of stiffened shells, Fig. 2;
- the experimental response of flat plates, Fig. 3.

In all the presented cases, the distorted similitudes, named *avatars*, are able to well represent the original response.

A possible list of the main open issues are here reported:

★ The experimental activities, which opened the collaboration between the Italian and Chilean institutions, evidenced the central role played by the different damping conditions and modal densities associated with the *avatars*.

★ Theoretical and numerical investigations are necessary for fluid-filled systems, where the ratio of the structural and acoustic natural frequencies should be kept too.

★ The sensitivity analysis on the main parameters remains to be investigated.

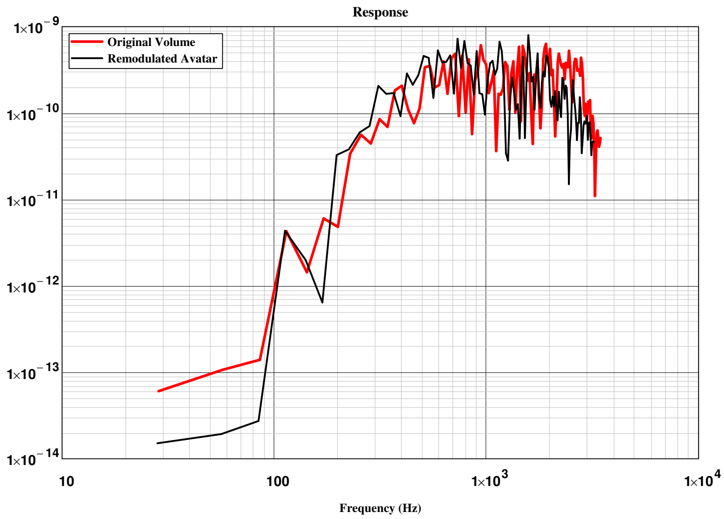

Figure 1: Local response of an acoustic volume (pressure [Pa] vs frequency [Hz]: Volume=$2.1 \times 3.1 \times 2.9 = 18.879m^3$; Modes, $NM = 30600$; scaling coefficients: $r_x = 0.3$, $r_y = 0.4$, $r_z = 0.5$.

This last point is the most important since it is related to the possibility to transform SAMSARA, *Similitude and Asymptotic Models for Structural-Acoustic Research and Applications*, in a real engineering tool. In fact, having demonstrated that the *avatars* exist, the conditions for getting bounded (and little interval) responses would be highly desired.

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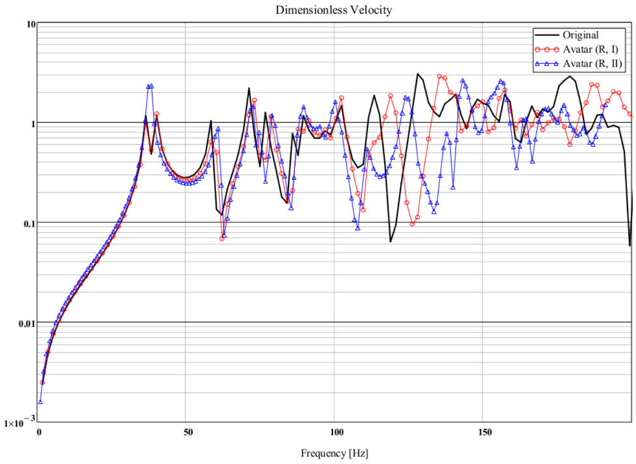

Figure 2: Forced response of stiffened cylindrical shells : $L = 10\,m$, $R = 2\,m$, $h = 1\,mm$; $r_L = r_R = 0.5$, $r_h = 1$; variation with the number of stiffeners, [6].

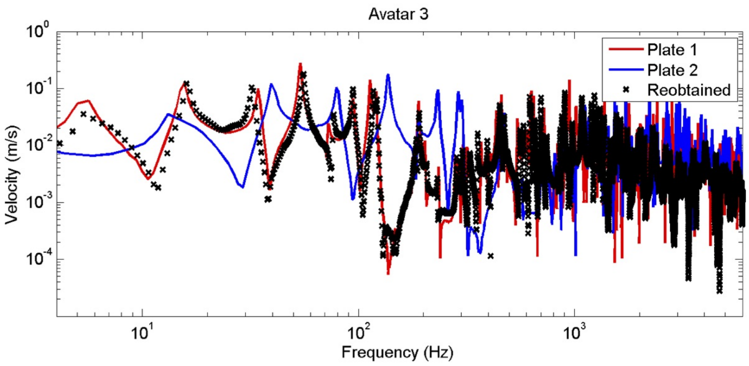

Figure 3: Structural experimental responses of flat plates, [7].

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Nonlinear Vibration of Belt-drive Systems with a Continuous Belt Model and Belt Flexural Rigidity

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Abstract

Belt-drive systems are common constituent elements in many rotational machine systems. In this work, the steady-state periodic response of a belt-drive system with two pulleys, an accessory pulley and a one-way clutch is presented. The belt-drive system is investigated under dual excitations for the first time. Specifically, the double excitations consist of harmonic vibration of the driving pulley and inertia excitation. The above and below belt spans are modeled as axially moving viscoelastic beams by considering belt flexural rigidity. Therefore, two nonlinear integro–partial–differential equations are established for governing the transverse vibrations of the above and below belt spans. Furthermore, the transverse vibrations of axially moving viscoelastic beams are coupled with the angular vibrations of the pulleys by nonlinear dynamic tension. Angular vibrations of the driven pulley and accessory are modeled as coupled piecewise ordinary differential equations for describing the unidirectional decoupling function of the one-way clutch. In order to eliminate the influence of the belt flexural rigidity on the boundaries, the non-trivial equilibriums of the belt-drive system are numerically studied. Furthermore, nonlinear piecewise discrete-continuous equations are derived by introducing a coordinate transform. Coupled vibrations of the belt-drive system are investigated via the Galerkin method. The resonance regions of the belt spans, the driven pulley and the accessory pulley are described. By comparing the numerical results with and without one-way clutch, significant damping effect of clutch on the stable steady-state response is discovered. Moreover, the influences of the amplitude of the motion of the driving pulley and the motion of the foundation are investigated. Furthermore, results illustrate that the two excitations interact on the amplitude-frequency curves, as well as the damping effect of the one-way clutch.

1. Introduction

Kong and Parker confirmed that the bending stiffness of the belt has important influences. Neglecting it may cause significant errors in vibration characteristics of the belt-drive system [1]. In another aspect, by considering the bending stiffness and the equilibrium, Wang and Mote found that significant error in the predicted vibration characteristics may occur if the coupling between vibration of the belt spans and pulleys is neglected [2]. One more thing should be noted, the viscoelasticity had been confirmed as an important characteristic of axially moving belt. Although the coupled vibration of belt-drive systems has been widely studied, researchers always focused the dynamics under a single excitation [3]. However, the damping effect of the one-way clutch on the dynamic system under multiple excitations has not been addressed. In this work, these elements, bending stiffness and the viscoelasticity of the belt, the equilibrium of the system, the coupling between the belt and pulleys, and the coupling between different excitation sources, are all involved.

2. Equations of Motion

As shown in Figure 1, a two-pulley belt-drive model coupled with accessory pulley is established. The motion of the foundation of the pulley-belt system is expressed as $B\cos(\omega_f t)$, where B and ω_f are the amplitude and the frequency, t is time. The two pulleys are assumed with the same radius r and rotational inertia J. $\theta_1(t)$ and $\theta_2(t)$ are the angular displacements. M is the pre-load. $\theta_a(t)$ and J_a are the angular displacements and the rotational inertia of the accessory, respectively. The driven pulley and the accessory are elastically connected with a wrap spring, with stiffness K_d. c, ρ, E, and A, respectively, are speed, density, Young's modulus and cross-sectional area of the moving belt, and all are assumed to be constant. x_1 and x_2, respectively, are the neutral axis coordinate of belt span 1 and 2. $w_1(x_1,t)$ and $w_2(x_2,t)$ are the transverse displacements as well. P_i (i=1,2) are the axial tension of the belt span 1 and 2, respectively.

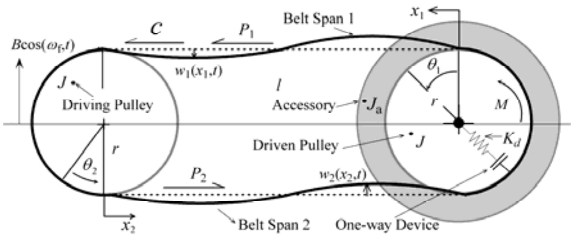


Figure 1. Belt-drive system consists of two pulleys, accessory pulley and a one-way clutch.

The equations of the motion of the system are obtained from Newton's second law as

$$\begin{aligned}
&\rho A\left(w_1,_{tt}+2c\,w_1,_{x_1 t}+c^2 w_1,_{x_1x_1}\right)+B\rho A\omega_f^2\cos(\omega_f t)\\
&\quad=\left[P_0+\frac{EAr}{l}(\theta_2-\theta_1)+\frac{EA}{2l}\int_0^l w_1,_{x_1}^2\,\mathrm{d}x_1\right]w_1,_{x_1x_1}-\left(E+\alpha\frac{\partial}{\partial t}\right)Iw,_{x_1x_1x_1x_1}\\
&\rho A\left(w_2,_{tt}+2c\,w_2,_{x_2 t}+c^2 w_2,_{x_2x_2}\right)-B\rho A\omega_f^2\cos(\omega_f t)\\
&\quad=\left[P_0+\frac{EAr}{l}(\theta_1-\theta_2)+\frac{EA}{2l}\int_0^l w_2,_{x_2}^2\,\mathrm{d}x_2\right]w_2,_{x_2x_2}-\left(E+\alpha\frac{\partial}{\partial t}\right)Iw,_{x_2x_2x_2x_2}\\
&J\ddot{\theta}_2+c_b\dot{\theta}_2=\frac{EA}{2l}\left(4r(\theta_1-\theta_2)+\int_0^l w_2,_{x_2}^2\,\mathrm{d}x_2-\int_0^l w_1,_{x_1}^2\,\mathrm{d}x_1\right)r,\\
&J\ddot{\theta}_1+c_b\dot{\theta}_1=\frac{EA}{2l}\left(4r(\theta_2-\theta_1)+\int_0^l w_1,_{x_1}^2\,\mathrm{d}x_1-\int_0^l w_2,_{x_2}^2\,\mathrm{d}x_2\right)r+M-f(\delta\theta)K_d(\delta\theta),\\
&J_a\ddot{\theta}_a+c_a\dot{\theta}_a=-M+f(\delta\theta)K_d(\delta\theta)
\end{aligned} \tag{1}$$

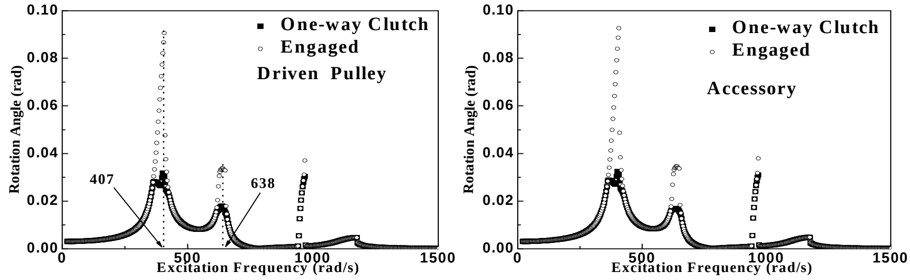
where P_0 is the initial axial tension, α is the dynamic viscosity, I is the moment of inertial and the comma preceding t or x_i denotes partial differentiation with respect to t or x_i. The dot denotes differentiation with respect to time and c_b is the damping coefficient between the belt and the pulley and c_a is the torsion damping coefficient of the accessory, $\delta\theta=\theta_1-\theta_a$ is the relative angular displacement, and $f(\delta\theta)$ is defined by

$$f(\delta\theta)=\begin{cases}1 & \delta\theta>0\\ 0 & \delta\theta\le 0\end{cases}. \tag{2}$$

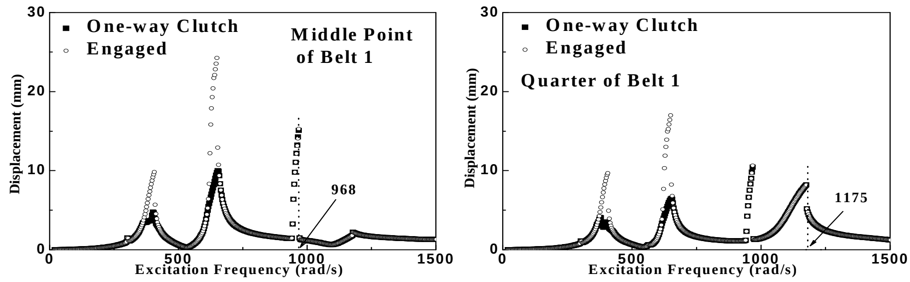
The boundary conditions are

$$w_i(0,t)=w_i(l,t)=0,\ w_i,_{x_ix_i}(0,t)=w_i,_{x_ix_i}(l,t)=1/r,\ (i=1,2)\,. \tag{3}$$

By using Galerkin' method, one can obtain the stable steady-state responses. Numerical results in Figures 2 and 3 found that the two excitations, caused by the rotation of the driving pulley and the rectilinear motion of the foundation, may cancel each other out when the two excitation frequencies equal. Furthermore, the one-way device efficiently reduces the resonance responses of the rotation vibration and transverse vibration of the pulley-belt dynamic system. Moreover, the damping effect of the one-way clutch impacts all resonance peaks of the angular vibrations and rectilinear vibrations.

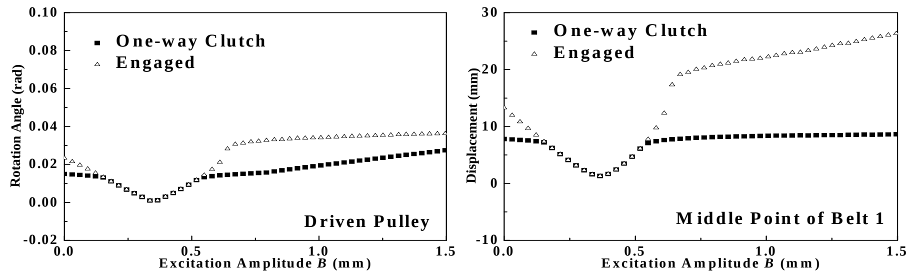


(a) the rotation of the driven pulley (b) the rotation of the accessory



(c) the vibration of the belt span 1 (d) the vibration of belt span 1

Figure 2. The sweep frequency response of the system with and without one-way device: ω=ω_f



(c) the rotation vibration: ω=638 (d) the transverse vibration: ω=638

Figure 3. The effects of the amplitude of the inertia excitation on the response: ω=ω_f

Acknowledgments

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Asymptotic Modal Analysis for Interconnected Systems: Discrete and Continuous Systems and the Effect of Nonlinearity

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Abstract

Several years ago, the present author and his colleague, D. Tang, addressed some of these issues in the context of a prototypical system where the primary system was an elastic plate with a discrete spring/mass attached. The plate is responding in the limit of a large number of modes in a chosen frequency bandwith [1]. In the present paper, generalizations and further verification of the results of [1] are considered.

Asymptotic Modal Analysis (AMA) and its predecessor and inspiration, Statistical Energy Analysis (SEA), seek to provide a basis for modeling and understanding the response of dynamical systems which have many modes (degrees of freedom) in a frequency bandwidth of interest. There is a very substantial literature on SEA including the classic text [2] by one of the founders, Richard Lyon, and as subsequently revised and expanded upon by Lyon and Dejong [3]. SEA in turn was inspired by Statistical Mechanics and the treatment of large numbers of particles that has had such a profound impact on thermodynamics, fluid mechanics and continuum physics in general. One of the central pillars of SEA is the notion of equipartition of energy among the dynamical modes and the emphasis on modeling of a single global scalar of a system, i.e. its total energy, versus a detailed modeling of the spatial as well as temporal distribution of energy and displacement which is more typical of classical modal analysis. Indeed in SEA it is assumed that the energy in a system responding in many modes is uniformly distributed in space as a consequence of equipartition of energy among the modes.

AMA was developed to see if one could in fact derive this result or hypothesis in SEA from classical modal analysis by considering the limit (asymptote) of classical modal analysis when many modes are responding in a certain frequency bandwidth. The term, asymptotic, is used to emphasize that while the number of modes responding must be large they also must share similar properties such as resonant frequencies and damping so that the individual modal values may be replaced with average values such as the center frequency and associated damping.

One of the central findings of AMA is that there are certain points or lines in the spatial domain of the system where in the AMA/SEA limit the response is notably higher even though most of the system response is spatially uniform. To the present author's knowledge these special points and lines were first observed by the eminent scholar, Stephen Crandall and his students [4]. Subsequently the present author and his colleagues reproduced and expanded upon Crandall's discovery in the context of AMA.

In the present paper we build upon the prior AMA results in two ways. First the effect of nonlinearities will be considered. Most, if not all, of the prior literature on AMA and SEA deals with linear dynamical systems. Secondly one of the greatest challenges for both AMA and SEA is how to treat the interaction between two or more systems. The work in [1] was a first step using AMA to address this issue. The present paper is a next step. Treating the interaction of two or more systems in SEA has also proven to be a challenge, although some authors are optimistic that these issues have been largely resolved within the framework of SEA per se [5]. The present author will not comment on this assertion here other than to say that the present results are believed to reveal some of the complexities that must be addressed in AMA and possibly in SEA when the interaction of two or more dynamical systems is considered.

Inter alia the opportunity also will be taken to provide some deeper insight into the accuracy and range of applicability of the AMA results previously obtained in [1].

For an overview of some of the key results of AMA the reader may wish to see [6].

Figure 1 shows the relevant configuration considered in [1] and the present paper. Figure 2 shows another configuration that will also be considered in the present paper. Fig. 1 displays a continuum system (the elastic plate) connected to a discrete system, the spring/mass. Figure 2 shows two continuum systems (elastic plates or beams) connected by a rigid member, the vertical rod.

For the configuration shown in Fig. 1, in the present paper the spring will become nonlinear, rather than the linear spring of [1], and also some of the subtleties of what happens when the spring/mass resonance frequency occurs with the frequency bandwidth of response will be considered. For the configuration shown in Figure 2, the focus will be on how the force of excitation on one plate is transmitted to the second plate.

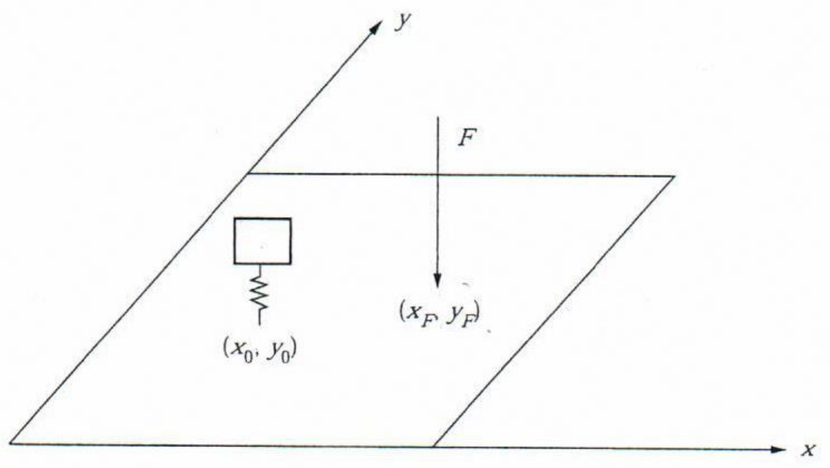

Figure 1. Geometry of the plate structure with concentrated mass/spring system.

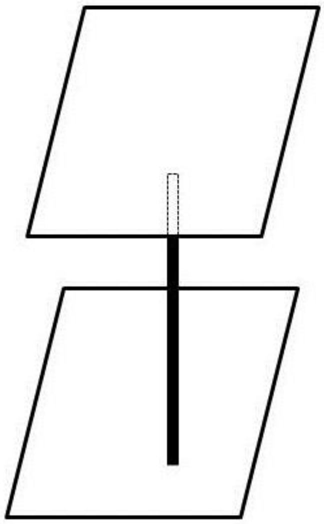

Figure 2. Two plates connected by a vertical rod.

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Damped vibrations of plates with viscoelastic treatments: analytical and Ritz solutions based on hierarchical layerwise models

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Introduction and contents. Passive treatments involving viscoelastic materials can be adopted as an effective means of reducing vibrations and noise radiation from light and stiff plate-like structural components. Typical configurations involve surface bonded or embedded damping treatments [1]. It is known that viscoelastic laminated structures are characterized by highly deformed patterns inside the viscoelastic layers and high deformation discontinuities arising at the interfaces between the soft viscoelastic and the stiff elastic layers. Therefore, accurate modal estimates for optimal design of viscoelastic treatment configurations can be obtained only by an accurate representation of the through-the-thickness variation of the strain and stress fields. Due to the above needs, free vibration models of viscoelastic laminated plates relying on higher-order 2-D layerwise kinematic theories are developed in this work. In particular, since an optimal performance of damping treatments is typically achieved by analyzing many alternative configurations comprising arbitrary stacking sequences of a single or multiple kinds of viscoelastic materials, design-tailored analytical and semi-analytical models are derived which can be efficiently used for preliminary parametric and/or optimization studies. The formulation proposed by Carrera [2] is exploited to derive invariant layer- and order-independent nuclei of the frequency-dependent complex stiffness and mass matrices. The nuclei are then properly expanded and assembled to yield the final matrices of the non-linear eigenvalue problem governing the damped vibrations of the multilayered plate. The resulting plate models are hierarchical in nature and provide great flexibility to the designer in the through-the-thickness approximations for arbitrary stacking sequences of both thin and thick elastic and viscoelastic layers.

Model description. The present formulation refers to a freely vibrating rectangular plate of side lengths a and b, and total thickness h. The plate consist of N_ℓ layers, which can be made of orthotropic elastic or frequency-dependent viscoelastic material in isothermal conditions. The kth layer has thickness h_k and is located between z_k and z_{k+1} in the thickness direction. According to the elastic-viscoelastic correspondence principle, the stress-strain relation in viscoelastic layers is expressed in the frequency-domain through the complex modulus approach as $\sigma^k = \mathbf{C}^k(j\omega)\varepsilon^k$, where $C^k_{ij}(j\omega) = C'^k_{ij}(\omega) + jC''^k_{ij}(\omega)$. By introducing the loss factor $\eta^k_{ij}(\omega)$ as the ratio between the frequency dependent loss modulus $C''^k_{ij}(\omega)$ and the frequency-dependent storage modulus $C'^k_{ij}(\omega)$, each complex modulus $C^k_{ij}(j\omega)$ can be written as $C^k_{ij}(j\omega) = C'^k_{ij}(\omega)\left[1+\eta^k_{ij}(\omega)\right]$. Higher-order 2-D kinematic theories are employed by expressing the displacement vector for the k-th layer as $\mathbf{u}^k(x,y,\zeta_k) = \sum_\tau F_\tau(\zeta_k)\mathbf{u}^k_\tau(x,y) = F_\tau(\zeta_k)\mathbf{u}^k_\tau(x,y)$, where $\mathbf{u}^k_\tau = \left\{\begin{array}{ccc} u^k_\tau & v^k_\tau & w^k_\tau \end{array}\right\}$ is the vector of kinematic variables, $\tau = t,r,b,\ r = 2,\dots,N-1,\ \zeta_k$ is the local dimensionless layer coordinate $(-1 \le \zeta_k \le 1)$, N is the order of the theory, and $F_\tau(\zeta_k)$ are assumed thickness functions, which are taken as a combination of Legendre polynomials in order to satisfy the interlaminar continuity of the displacements. In-plane (p) and out-of-plane (n) components of the strain vector are linearly related to displacements by $\varepsilon^k_\mathrm{p} = \mathscr{D}_\mathrm{p}\mathbf{u}^k = F_\tau\mathscr{D}_\mathrm{p}\mathbf{u}^k_\tau$ and $\varepsilon^k_\mathrm{n} = \mathscr{D}_\mathrm{n}\mathbf{u}^k + \partial\mathbf{u}^k/\partial z = F_\tau\mathscr{D}_\mathrm{n}\mathbf{u}^k_\tau + F_{\tau,z}\mathbf{u}^k_\tau$, where $\mathscr{D}_\mathrm{p}$ and $\mathscr{D}_\mathrm{n}$ are matrices of in-plane differential operators and $F_{\tau_{,z}} = \mathrm{d}F_\tau/\mathrm{d}z$. According to the above framework, the principle of virtual displacement in the absence of any external load can be written as

$$\sum_{k=1}^{N_\ell} \int_{\Omega_k} \int_{z_k}^{z_{k+1}} \left[\delta \left(\mathscr{D}_\mathrm{p} F_\tau \mathbf{u}_\tau^k \right)^\mathrm{T} \sigma_\mathrm{p}^k + \delta \left(\mathscr{D}_\mathrm{n} F_\tau \mathbf{u}_\tau^k \right)^\mathrm{T} \sigma_\mathrm{n}^k + \delta \left(F_{\tau,z} \mathbf{u}_\tau^k \right)^\mathrm{T} \sigma_\mathrm{n}^k \right] \mathrm{d}\Omega\, \mathrm{d}z = -\sum_{k=1}^{N_\ell} \int_{\Omega_k} \int_{z_k}^{z_{k+1}} \delta \left(F_\tau \mathbf{u}_\tau^k \right)^\mathrm{T} \rho^k F_s \ddot{\mathbf{u}}_s^k \,\mathrm{d}\Omega\, \mathrm{d}z \quad (1)$$

where δ denotes the virtual operator, σ_p^k and σ_n^k are the in-plane and normal components of the stress vector, respectively, Ω_k is the middle surface domain of the k-th layer and ρ^k is the mass density. Integration by parts and arbitrariness of virtual variations yield the following boundary value problem

$$\mathbf{L}_{\tau s}^k(\omega)\mathbf{u}_s^k + \rho^k J_{\tau s}^k \ddot{\mathbf{u}}_s^k = \mathbf{0} \quad \text{over} \quad \Omega_k \qquad \mathbf{B}_{\tau s}^k(\omega)\mathbf{u}_s^k = \mathbf{0} \quad \text{on plate boundary} \qquad (2)$$

where $\mathbf{L}_{\tau s}^k(\omega)$ and $\mathbf{B}_{\tau s}^k(\omega)$ are 3×3 frequency-dependent nuclei of differential operators.

The Navier solution. Damped free vibration Navier solutions are obtained for fully simply-supported plates with layers made of specially orthotropic material ($C_{16}^k = C_{26}^k = C_{36}^k = C_{45}^k = 0$). The kinematic unknowns of the layerwise model are expressed as the following Fourier series

$$\mathbf{u}_\tau^k = \left\{ \begin{array}{l} u_{\tau mn}^k \cos(m\pi x/a) \sin(n\pi y/b) \\ v_{\tau mn}^k \sin(m\pi x/a) \cos(n\pi y/b) \\ w_{\tau mn}^k \sin(m\pi x/a) \sin(n\pi y/b) \end{array} \right\} e^{j\omega t} \quad (m,n = 0,1,2,\dots) \qquad (3)$$

which identically satisfies the boundary conditions in Eq. (2) for simply-supported edges. The expansion in (3) is substituted into equations of motion (2) and, since the resulting equations must hold for every point (x,y) over each Ω_k, the following eigenvalue problem at layer-level is obtained $\left[\mathbf{K}_{\tau smn}^k(\omega) - \omega^2 \mathbf{M}_{\tau smn}^k\right] \mathbf{u}_{smn}^k = \mathbf{0}$, which is valid for any theory-related index-pair (τ,s) and any layer k. The above 3×3 stiffness and mass nuclei $\mathbf{K}_{\tau smn}^k$, $\mathbf{M}_{\tau smn}^k$ are then expanded by varying τ and s. The resulting matrices are assembled through the layers by imposing the continuity at each layer interface and the Navier solutions for the (m,n) mode of the multilayered plate are obtained by solving the non-linear eigenvalue problem $\left[\mathbf{K}_{mn}(\omega) - \omega^2 \mathbf{M}_{mn}\right] \mathbf{u}_{mn} = \mathbf{0}$.

The Ritz solution. For viscoelastic laminated plates with arbitrary lamination lay-up and various boundary conditions (combination of free, clamped and simply-supported edges), a Ritz solution at any layer k is sought in the following form

$$\mathbf{u}_\tau^k = \left\{ \begin{array}{c} N_{u\tau i}(x,y) u_{\tau i}^k \\ N_{v\tau i}(x,y) v_{\tau i}^k \\ N_{w\tau i}(x,y) w_{\tau i}^k \end{array} \right\} e^{j\omega t} \quad (i = 1,2,\dots,M) \qquad (4)$$

where $N_{\alpha\tau i}$ ($\alpha = u,v,w$) are admissible functions given by the product of geometrically-compliant boundary functions and one-dimensional Chebyshev polynomials [3]. The approximation in Eq. (4) is directly used into the PVD statement Eq. (1). The arbitrariness of the virtual variations yields the eigenvalue problem $\left[\mathbf{K}_{\tau sij}^k(\omega) - \omega^2 \mathbf{M}_{\tau sij}^k\right] \mathbf{u}_{sj}^k = \mathbf{0}$, which is valid for any pair (τ,s), any Ritz-related index-pair (i,j) and any layer k. The above Ritz nuclei are first expanded according to the order of the theory. Enforcing the interlaminar continuity conditions yields the multilayer matrices, which are finally expanded again through the variation of the Ritz-related indices i and j. The resulting non-linear eigenvalue problem takes the form $\left[\mathbf{K}(\omega) - \omega^2 \mathbf{M}\right] \mathbf{u} = \mathbf{0}$.

Illustrative results. Two examples are here presented to validate the present formulation and illustrate its capabilities. The first example in Table 1 shows that first-order layerwise models (LD1) cannot be enough to provide accurate vibration and damping estimates, and higher-order theories (LD2 in this case) can be required. From the example in Table 2 it is confirmed that equivalent single-layer theories, even those based on higher order deformations [5], are not suitable to describe with adequate accuracy the dynamic behavior of multilayered plates with embedded viscoelastic layers. More comparison and benchmark results will be presented at the symposium. In particular, the effect of various parameters on the damped vibrations of viscoelastic laminated plates will be discussed, including different boundary conditions, stacking sequences, and location and numbers of damping layers.

Table 1: Modal frequencies and loss factors of a three-layer symmetric sandwich simply-supported plate ($a = 0.3048$ m, $b = 0.3480$ m). Isotropic faces: $h_1 = h_3 = 0.762$ mm, $E_1 = E_3 = 6.89 \times 10^{10}$ Pa, $\nu_1 = \nu_3 = 0.3$, $\rho_1 = \rho_3 = 2740$ kg/m^3. Isotropic core with frequency-independent viscoelastic properties: $h_2 = 0.254$ mm, $\rho_2 = 999$ kg/m^3, $G_2 = 0.896 \times 10^6$ Pa, $\eta_2 = 0.5$.

(m,n)	Ref. [4]		Navier (LD1)		Navier (LD2)	
	f (Hz)	η	f (Hz)	η	f (Hz)	η
(1,1)	60.2	0.190	62.3	0.178	60.2	0.190
(1,2)	115.2	0.203	120.9	0.185	115.2	0.203
(2,1)	130.2	0.199	137.3	0.180	130.4	0.199
(2,2)	178.5	0.181	189.5	0.160	178.5	0.181
(1,3)	195.4	0.174	208.0	0.153	195.4	0.174

Table 2: Modal frequencies and loss factors of a square $[0/90/0/core/0/90/0]$ simply-supported plate with $a/h = 10$, $h_c/h = 0.94$, carbon FRP faces and isotropic core with frequency-dependent viscoelastic material (properties are taken from Ref. [5]).

Mode	Ref. [5]		Navier (LD3)		Ritz (LD1)	
	f (Hz)	η (%)	f (Hz)	η (%)	f (Hz)	η (%)
1	117.65	25.37	107.22	27.58	107.23	27.60
2	198.28	27.79	179.80	29.44	179.89	29.44
3	201.25	27.64	180.97	29.61	181.06	29.61
4	257.68	28.05	232.65	29.73	232.82	29.72
5	291.97	28.26	262.21	29.90	262.94	29.90

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A Comparison of Acoustic and Mechanical Excitation for Damping Estimation in Structural Panels

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Methods abound for structural damping estimation relevant to noise transmission in plates and shells. Differences in methods are noted with regard to excitation type, typically mechanical with either random or sinusoidal waveforms, as well as the parameter extraction method used—the most popular of which are the Power Injection Method (PIM), the Impulse Response Decay Method (IRDM) [1] and the Random Decrement Technique (RDT) [5]. For these three methods, numerous process variables, including excitation location, response measurement location, averaging method and others have been studied by Ewing and Dande for free-hanging, mechanically-excited plates [2,3]. The current effort is focused on the comparison of acoustic and mechanical excitation strategies for panel damping loss measurements. For this comparison, a carbon fiber-epoxy panel stiffened with 5 "hat" stiffeners of the same material is used.

Experimental Setup

A 4-foot by 8-foot graphite-epoxy stiffened panel from Spirit Aerosystems was used for this research. The drive signal for both types of excitation was a pseudo-random waveform which was filtered by an adjustable notch filter unit into 1/3 octave frequency bands. Here only the 500Hz and 1000Hz center frequency bands will be discussed.

Figure 1. Stiffened panel used for mechanical (left) and acoustic (center and right) excitation.

Mechanical excitation was provided by a small, 2-lb force electrodynamic shaker. For mechanical excitation, the location of the excitation points is often problematic: one gets very poor estimates if mechanical excitation is applied along a node line of one or more modeshapes in a frequency band of interest.

Acoustic excitation was ultimately provided by a 4-inch diameter speaker located at a distance of approximately 8 inches from the panel. Initially, a 12-speaker, duodecahedron assembly was used to provide diffuse acoustic excitation, as this excitation is readily-available in most acoustic transmission loss facilities, including the Great Plains Acoustic Test Facility at the University of Kansas. This diffuse field, as expected, was ineffective at exciting panel modal response enough to allow damping estimation with available algorithms. A more effective way to get sufficient amplitudes in response, is to apply acoustic energy from the speaker to an area the size of the characteristic *feature size* in the modal response of the structure. In this context, the feature size is the size of an area of a structure which shows in-phase response throughout during a cycle of vibration. The speaker should be close enough to the structure that the pressure pulses from the speaker diaphragm are essentially in-phase as they impact a nearby structure. This suggests a ratio of the speaker diameter to the distance to the structure to be in the range of 1:1 to 1:2. In these tests, the ratio was approximately 1:2.

For both excitation types, the response measurement points were identical. Two were on stringers and two were on skin between stringers. Accelerometers were used to represent the motion of the panels.

Experimental Data

Using mechanical excitation, frequencies and mode shapes for one quadrant of the stiffened panel were determined using ME'scope software [4]. Figure 2 shows representative mode shape *amplitudes* for modes in the 500 and 1000 Hz full octave frequency bands. This view, in which a quadrant of the panel has the dimensions, 2 feet by 4 feet, clearly shows the range of "feature sizes" in the responses, that is, less than a foot. This indicates that the speaker size and distance from the test article were appropriate.

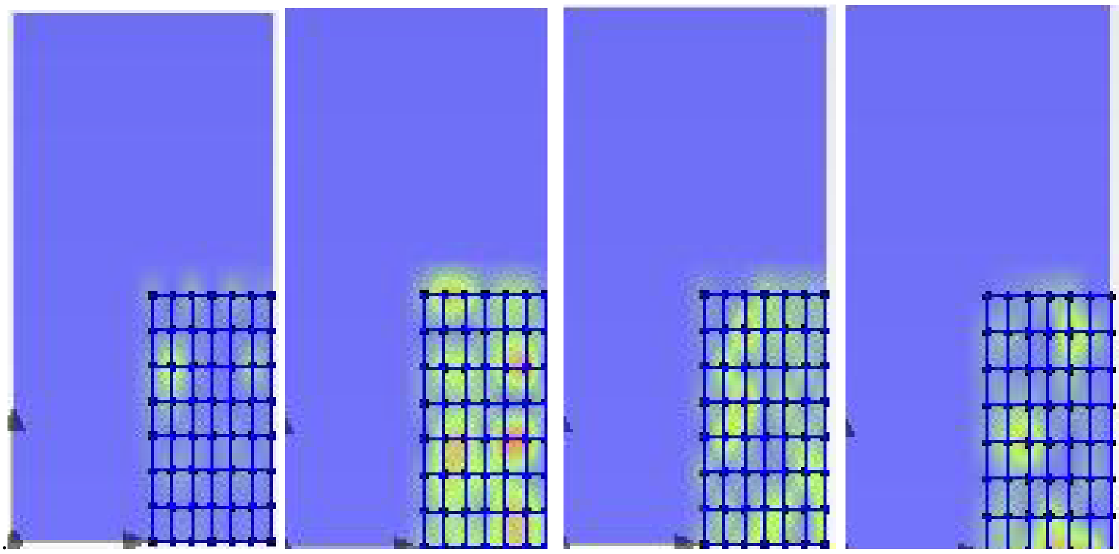

Figure 2. Mode shape amplitudes for modes in the 500 and 1000 full octave frequency ranges: from left to right, the modes appear at 460 Hz, 559 Hz, 983 Hz and 1080 Hz.

Loss factor estimation

During data acquisition for the loss factor estimations, the acceleration at the 4 response measurement points were recorded for time periods dependent on the frequency band of interest, typically on the order of 10-20 seconds. Loss factors were then estimated with the RDT algorithm. In particular, the selectively triggered, time-domain response (acceleration) for 4 "representative" points on the panel were averaged to obtain a decaying sinusoid. The decay rate for each of these locations was determined using, based on earlier research, two types of averaging: directly averaging (DA); and, averaging the positive branch of the autocorrelation of the signal (AA). Figure 3 shows the ***average*** loss factor for the 4 response locations as a function of excitation type and averaging type. The loss factor estimates for mechanical excitation, however, showed a decidedly larger standard deviation in the estimates.

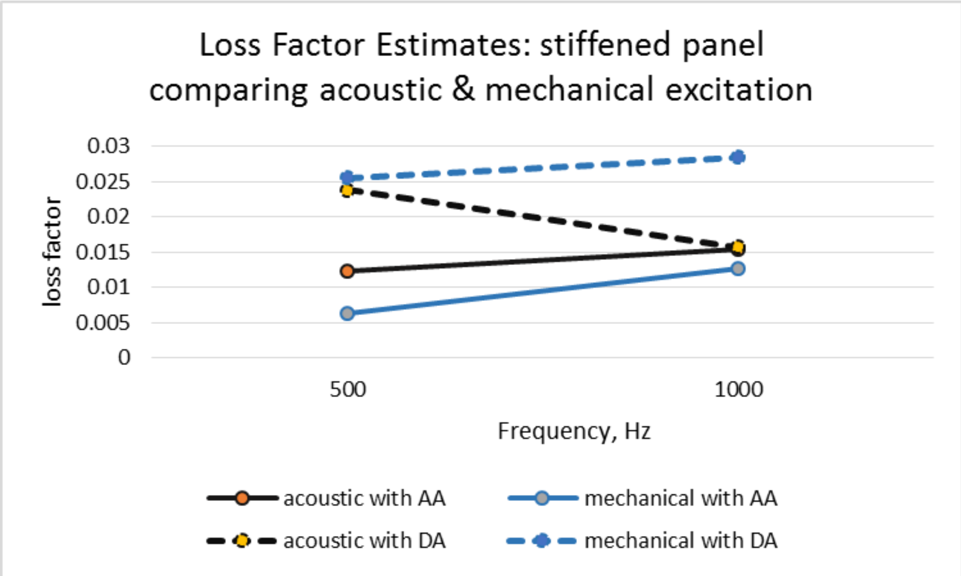

Figure 3. Loss factor estimates for the stiffened panel.

The following observations can be made:

- For averaging using the autocorrelation of the averaged response (AA), the acoustic and mechanical excitation give rather similar damping losses.
- For direct averaging (DA) the loss factors are higher than using autocorrelation averaging (the opposite of what was found in a study of free-hanging uniform aluminum panels).
- The acoustic excitation method resulted in damping loss estimates which had a significantly lower standard deviation, that is, less scatter than when mechanical excitation is used.

The latter observation leads one to believe that acoustic excitation may allow one to be less concerned about the location of the speaker when using acoustic excitation, whereas there is significant concern when using mechanical excitation.

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On the Effect of Damping in Continuous Nonconservative Mechanical Systems

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Abstract

Many systems in mechanical engineering exhibit self-excited vibrations, which have to be suppressed or completely avoided by different design approaches. A common method to avoid self-excited vibrations is to add damping to the system with the intent to stabilize it. This may however also have the opposite effect, namely destabilization, as has been known for a long time [1, 2]. More recently it was shown in [3] that weakly stable discrete systems of the **M**-**K**-**N** type can always be destabilized by adding an infinitesimal small damping matrix. It was also shown that the common assumption of RAYLEIGH damping, i.e. writing $\mathbf{D} = \alpha\mathbf{M} + \beta\mathbf{K}$ is highly problematic, since the choice of α and β, however small, can be decisive for the stability. Furthermore, in [4] the destabilizing effect of damping is shown for different practical mechanical engineering systems subject to nonconservative forces, which can orginate in physical systems, for example, from frictional contact forces, aerodynamic coupling or other energy sources. In the following, the destabilizing effect of damping is analyzed for continuous mechanical systems without using any discretization method. Instead of the symmetric matrices **M** and **K** and the skew-symmetric matrix **N**, one now has self-adjoint and skew-adjoint operators, respectively.

In the following we consider the mechanical system shown in Fig. 1. The system consists of an elastic ring which is in frictional contact with a rigid rotor. In addition, a particle with a spring is attached to the elastic ring in order to disturb the symmetry. The equations of motion are derived using HAMILTON's principle

$$\int_{t_1}^{t_2} (\delta\mathscr{L} - \delta\mathscr{W}_f - \delta\mathscr{W}_d)\,\mathrm{d}t = 0 \quad \text{with} \quad \delta\mathscr{L} = \delta\mathscr{T} - \delta\mathscr{U}\,, \tag{1}$$

where $\mathscr{T}$ and $\mathscr{U}$ are the kinetic and potential energy of the elastic ring and the lumped element, $\mathscr{W}_f$ the external work done by the frictional layer and $\mathscr{W}_d$ the work done by damping forces. The corresponding expressions[1] are

$$\begin{aligned}
\delta\mathscr{T} &= \int_0^L \rho A\dot{w}\delta\dot{w}\,\mathrm{d}x + m\dot{w}(0,t)\delta\dot{w}(0,t)\,, \qquad \delta\mathscr{U} = \int_0^L EIw''\,\delta w''\,\mathrm{d}x + kw(0,t)\delta w(0,t)\,,\\
\delta\mathscr{W}_f &= \int_0^L \left((k_f w + d_f\dot{w})\delta w - \mu\frac{h}{2}(k_f w + d_f\dot{w})\delta w' \right) \mathrm{d}x\,,\\
\delta\mathscr{W}_d &= \alpha\left(\int_0^L \rho A\dot{w}\delta w\,\mathrm{d}x + m\dot{w}(0,t)\delta w(0,t)\right) + \beta\left(\int_0^L EI\dot{w}''\,\delta w''\,\mathrm{d}x + k\dot{w}(0,t)\delta w(0,t)\right),
\end{aligned} \tag{2}$$

where $w(x,t)$ is the displacement of the ring, E the YOUNG's modulus, I the area moment of inertia, ρ the mass density, A, h and L the cross section, the height and the length of the ring, m

[1] The elastic ring is modeled by using EULER-BERNOULLI beam theory and the expressions describing the frictional contact are linearized.

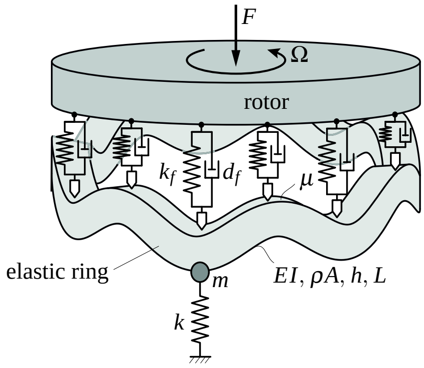


Figure 1: Mechanical model of an elastic ring in frictional contact with a rigid rotor.

the mass and k the stiffness coefficient of the lumped element, d_f and k_f the damping and stiffness coefficient of the friction layer and μ the friction coefficient. Furthermore, α and β are the mass-proportional and stiffness-proportional coefficients of the RAYLEIGH damping assumed for the system. Carrying out the variation with respect to the displacement $w(x,t)$ one obtains the partial differential equation

$$\rho A\ddot{w}+EIw''''+\alpha\rho A\dot{w}+\beta EI\dot{w}''''+d_f\dot{w}+k_f w+\mu\frac{h}{2}\left(d_f\dot{w}'+k_f w'\right)=0, \tag{3}$$

with the boundary conditions

$$\begin{aligned} &w(0,t)=w(L,t)\,, \quad w'(0,t)=w'(L,t)\,, \quad w''(0,t)=w''(L,t)\,, \\ &EI\left[w'''(0,t)-w'''(L,t)\right]=m\ddot{w}(0,t)+[\alpha m+\beta k]\,\dot{w}(0,t)+k\,w(0,t)\,. \end{aligned} \tag{4}$$

Inserting the *ansatz* $w(x,t)=\hat{w}\,\mathrm{e}^{\kappa x}\,\mathrm{e}^{\lambda t}$ into the partial differential equation (3) yields

$$\left[\rho A\lambda^2+EI\kappa^4+\alpha\rho A\lambda+\beta EI\lambda\kappa^4+d_f\lambda+k_f+\mu\frac{h}{2}\left(d_f\lambda\kappa+k_f\kappa\right)\right]\hat{w}\,\mathrm{e}^{\kappa x}\,\mathrm{e}^{\lambda t}=0\,. \tag{5}$$

The nontrivial solutions $\kappa_1,\dots,\kappa_4$ of Eq. (5) are used to obtain the general solution

$$w(x,t)=\left(C_1\mathrm{e}^{\kappa_1 x}+C_2\mathrm{e}^{\kappa_2 x}+C_3\mathrm{e}^{\kappa_3 x}+C_4\mathrm{e}^{\kappa_4 x}\right)\mathrm{e}^{\lambda t}\,, \tag{6}$$

which has to fulfill the boundary conditions (4), leading thereby to the characteristic equation

$$\sum_{i=1}^{4}\left((-1)^i\left[m\lambda^2+(\alpha m+\beta k)\lambda+k-EI\kappa_i^3(\mathrm{e}^{\kappa_i L}-1)\right]\prod_{j=1,j\neq i}^{4}(\mathrm{e}^{\kappa_j L}-1)\prod_{n=1,n\neq i}^{3}\;\prod_{p=n+1,p\neq i,n}^{4}(\kappa_n-\kappa_p)\right)=0\,. \tag{7}$$

The solution of this characteristic equation gives the eigenvalues λ. For the parameters

$$\begin{aligned} &EI=5600\,\mathrm{Nm}^2,\ \rho A=6.8\,\mathrm{kg/m},\ d_f=10^{-6}\,\mathrm{Ns/m},\ k_f=4\cdot 10^7\,\mathrm{N/m},\ \mu=0.3\,,\\ &h=0.02\,\mathrm{m},\ L=0.2\pi\,\mathrm{m},\ m=0.001\,\mathrm{kg},\ \alpha=0\,\mathrm{s},\ \beta=0\,\mathrm{s}^{-1},\ k=755.48\,\mathrm{kN/m} \end{aligned} \tag{8}$$

the imaginary and real parts of the first two eigenvalue pairs, which are the roots of the corresponding characteristic equation, are shown in Fig. 2 on the left-hand side for different values of the friction coefficient μ. Up to the critical value of $\mu_{\text{crit}}=0.3$ the roots have negative real parts, which are due to the parameter d_f only marginally below zero. For larger values of μ the system becomes unstable in the sense of self-excited vibrations. The effect of the damping coefficient β on the roots is highlighted in Fig. 2 on the right-hand side. Obviously increasing β has a destabilizing effect on one eigenvalue pair. For larger values than $\beta_{\text{crit}}=1.1\cdot 10^{-7}\,1/\mathrm{s}$ the system becomes stable again.

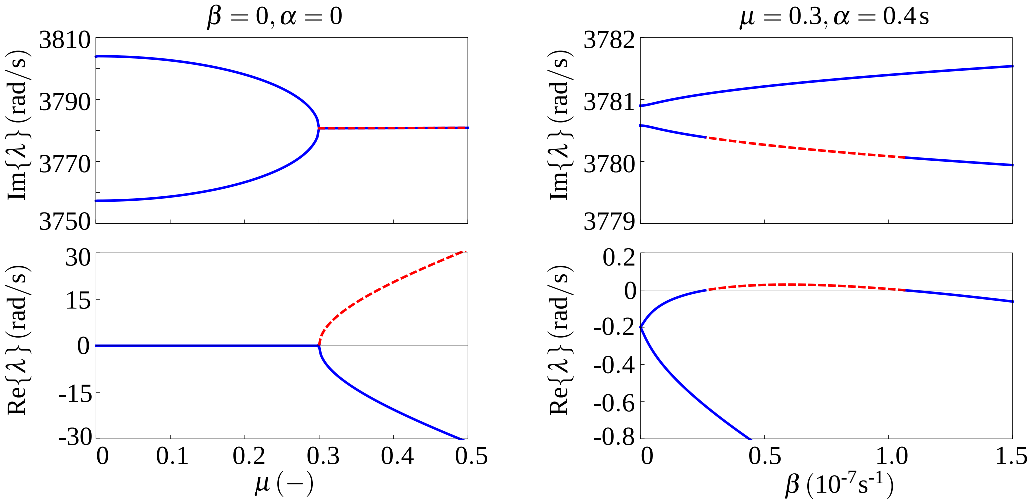
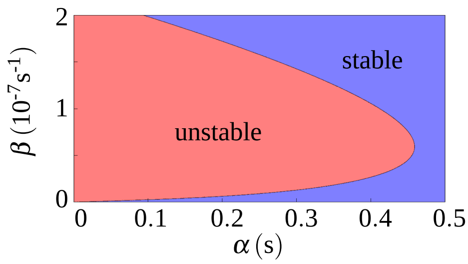


Figure 2: Imaginary and real parts of the first eigenvalue pair over the friction coefficient μ (left) and over the damping coefficient β (right).

Figure 3: Stability map for the first eigenvalue pair over the damping coefficients α and β.



In Fig. 3 the stability map for the first eigenvalue pair is shown over the damping coefficients α and β. It can be seen that for sufficiently large values of the damping coefficients the first eigenvalue pair always has negative real parts. The destabilizing effect of nonconservative continuous systems, shown here, exists only for relatively small damping values.

The introduced examples show that continuous nonconservative mechanical systems, which are asymptotically stable may be destabilized by adding damping to the system. It can be shown that also for continuous systems, the RAYLEIGH assumption is highly problematic in stability problems. It is of utmost importance that these effects be considered by designing damping for mechanical systems, which are subject to nonconservative effects.

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Energy Harvesting from Dual Cantilever Flutter

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Abstract

Many types of flow-induced vibration phenomena have been investigated over the past decades. Energy harvesting from these vibrations has been a growing topic of interdisciplinary research since the early works of Klakken *et al*, and Schmidt *et al*, (1983) [1,2]. The most popular flow-induced vibration energy harvesting techniques can be classified as either flutter or vortex-induced vibration (VIV). Energy harvesting from turbulence-induced vibration has also been shown to be quite effective [3].

This research presents a novel type of flow-induced vibration called dual cantilever flutter (DCF) which occurs between two similar adjacent cantilevers exposed to cross-flow. DCF flow-induced dynamics have similarities to both flutter and VIV; however, for the sake of simplicity, it has been categorized as a type of flutter. DCF was first witnessed while performing wind tunnel experiments on large arrays of lightly coupled flexible structures called *piezoelectric grass* – another novel flow-induced vibration energy harvesting method. It was observed that persistent resonant-like vibration occurred when two adjacent similar cantilevers were placed in cross-flow as illustrated in figure 1. These initial observational experiments showed that DCF vibration amplitude is highly dependent on both flow velocity U and the separation distance q.

At the onset of DCF both cantilevers become *locked* 180 degrees out of phase and their vibration amplitude increases steadily as flow velocity is increased. With a continued increase in flow velocity, the flutter amplitude approaches a maximum stable value and becomes less dependent on flow velocity. Frequency remains approximately constant over the entire velocity range.

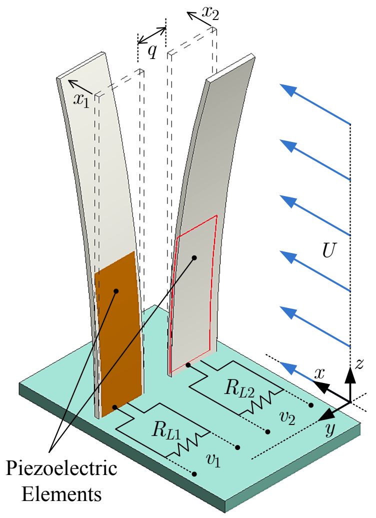

Figure 1: Schematic illustrating the dual cantilever flutter energy harvester concept in a unimorph configuration.

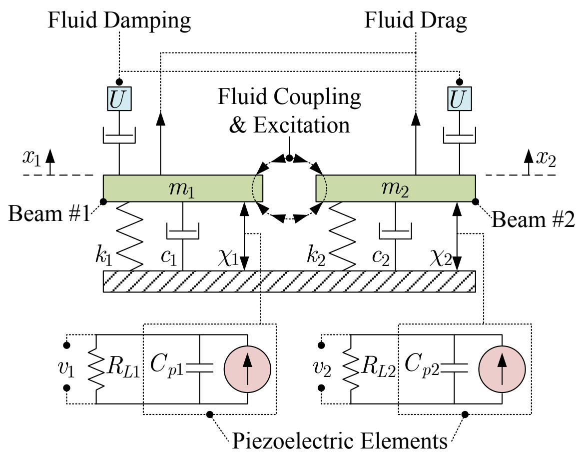

Figure 2: Equivalent lumped parameter representation of the dual cantilever harvester illustrating both fluid and electromechanical coupling.

Rather than attempting to model the complex fluid-structure interaction occurring along the length of the beams, a lumped parameter model based on the fundamental mode of each beam is shown to predict DCF dynamics quite well. The proposed model represents each beam as a single degree of freedom oscillator. Fluid drag, damping, and excitation terms are included along with both electromechanical and fluid coupling terms. Because the piezoelectric material is allowed to be shorter than the substrate as shown in figure 1, each beam has non-uniform mass and stiffness properties. These non-uniformities are accounted for by using a Rayleigh-Ritz model to estimate the equivalent mass m and stiffness k of the beams.

The general form of the lumped parameter coupled electromechanical differential equations for each beam can be represented as,

$$m\ddot{x} + c\dot{x} + kx + \chi v = \mathit{\Gamma}_c + \frac{1}{2}\rho A\left(C_{Dd}U^2 - C_{Dv}\dot{x}\left|\dot{x}\right|\right) + \beta(\theta^2 - x^2)\dot{x} \tag{1}$$

$$\dot{v} + \frac{1}{\tau}v = \varphi\dot{x} \tag{2}$$

where x is the tip displacement, m is the equivalent mass, c is viscous damping, k is the equivalent stiffness, χ and φ are electromechanical coupling terms, τ is the capacitive time constant, v is the time varying voltage across load resistance R_L, $\mathit{\Gamma}_c$ is the fluid coupling between the beams, ρ is the fluid density, A is the beam area normal to flow, C_{Dd} is the drag coefficient, C_{Dv} is the viscous drag coefficient, U is the cross-flow velocity, β is the linearity parameter, and θ is the stability threshold parameter. Single and double over-dots on x and v denote first and second time derivatives respectively. It is important to note that equations 1 and 2 represent the general form of the governing equations for a single beam; therefore, two sets of these equations (four total) are needed for two beams. Each piezoelectric element is modeled (equation 2) as a simple resistor-capacitor circuit with a strain-induced current source. The time constant can be defined as, $\tau = R_L C_p$ where R_L is the load resistance and C_p is the capacitance of the piezoelectric element. A schematic of this circuit model is shown in figure 2. Figure 2 illustrates a lumped parameter electromechanical system equivalent to the distributed parameter system shown in figure 1. Fluid coupling in this system is modeled using a nonlinear expression $\mathit{\Gamma}_c$ which is a function of separation distance q and both the relative displacement $(x_1 - x_2)$ and relative velocity $(\dot{x}_1 - \dot{x}_2)$ of each lumped mass. The fluid coupling is estimated using the following nonlinear expression,

$$\mathit{\Gamma}_{c(1,2)} = \frac{\gamma q^{2\alpha}\left(\dot{x}_1 \mp \dot{x}_2\right)\left|\dot{x}_1 - \dot{x}_2\right|}{\left[q^2 + \left(x_1 - x_2\right)^2\right]^{\alpha}} \tag{3}$$

where parameters γ and α are the velocity and displacement coupling terms respectively, and subscripts 1 and 2 correspond to beam 1 and 2. The second term on the right-hand side of equation 1 determines the fluid drag and damping forces. Notice that if $\dot{x} = 0$ and $U > 0$ then $v = 0$ and the only surviving terms predict the static drag deflection as simply $\rho A C_{Dd} U^2 / 2k$. The fluid excitation force is estimated with the third and final expression on the right-hand side of equation 1. This excitation term is adapted from the well-known Van der Pol model and can be described as a position-dependent damping term that can cause an oscillator to have a stable yet periodic solution called a limit cycle oscillation. When this position-dependent damping becomes a large enough negative value to overcome all damping in equations 1 and 2, then both coupled systems undergo limit cycle oscillations *i.e.*, the systems begin to flutter. For the results in figure 3, experimental data was used to update the fluid coupling and excitation parameters of the proposed model.

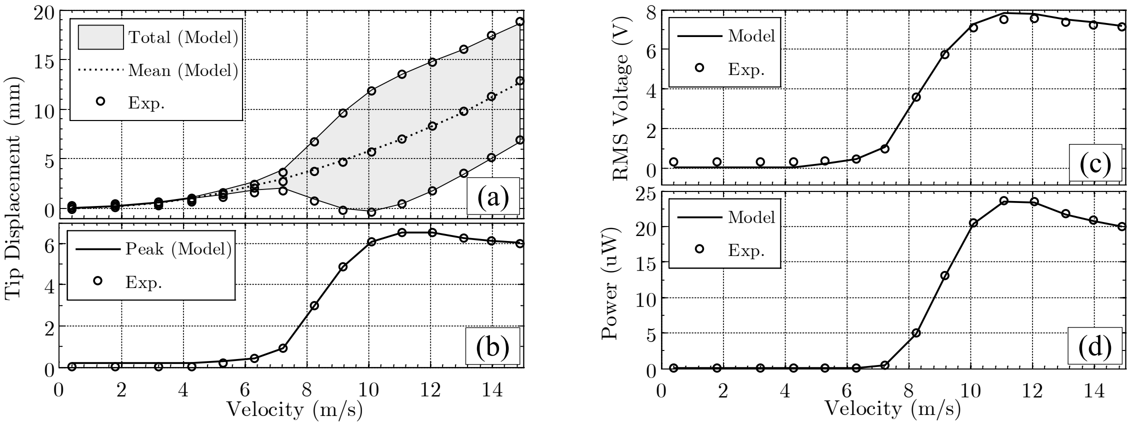

Figure 3: Model predictions and experimental measurements showing trends in a) total and mean tip deflection, b) peak tip deflection amplitude, c) RMS voltage, and d) power output as functions of flow velocity.

A series of wind tunnel experiments were performed both to proove the concept and to validate the proposed model. All results are sumarized in figure 3. The solid and dotted lines in figure 3a show the average flutter amplitude and mean deflection (respectively) of both beams as a function of flow velocity. As one would expect given the definition of drag force, the mean deflection increases as a function of U^2 . The flutter amplitude is approximately zero until ~7 m/s where an exponential increase ocurs until ~9 m/s where it then apears to tend toward a steady value. The gray region in figure 3a represents the total tip displacement envelope of each cantilever and the peak flutter amplitude is shown in figure 3b. Figures 3c-d show trends in voltage and power (respectively) corresponding to the flutter amplitude data given in figure 3b. Here, predictions of the updated model clearly agree very well with experimental data, thus validating the form of the proposed model given in equations 1 and 2.

In conclusion, this research highlights the discovery and initial modeling efforts of a flow-induced vibration phenomenon called dual cantilever flutter. A novel energy harvesting concept that takes advantage of DCF dynamics was also experimentally proven and successfully modeled using a lumped parameter approach. Results presented here have provided motivation to perform a more in-depth analysis of DCF and the proposed model. A primary goal of immediate future studies is to reveal the physical significance behind the fluid coupling and excitation terms and to determine the exact cause of this unique flow-induced vibration phenomenon. A deeper understanding of DCF dynamics will not only help maximize the efficiency of DCF energy harvesters, but may also help engineers avoid unwanted and potentially dangerous vibrations caused by DCF.

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Theoretical Basis of assembling structures in the Rayleigh Ritz Method

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Abstract

The use of negative structures in modelling cavities was discussed in previous symposia [1-3]. A formal proof that the use of negative structures to remove corresponding positive structural components in calculating natural frequencies has been established for discrete systems. This approach introduces extra spurious modes and methods of identifying and eliminating these modes have also been studied for the same. However, a corresponding proof for continuous systems, and its application poses some challenges. The focus of this paper is on these challenges.

To start with, let us consider a discrete *n* degree of freedom vibratory system A, which when connected to another *m* dof discrete system C^+ subject to *r* constraints that connect A and C^+ results in an $n+m-r$ dof system B. See Figure 1.

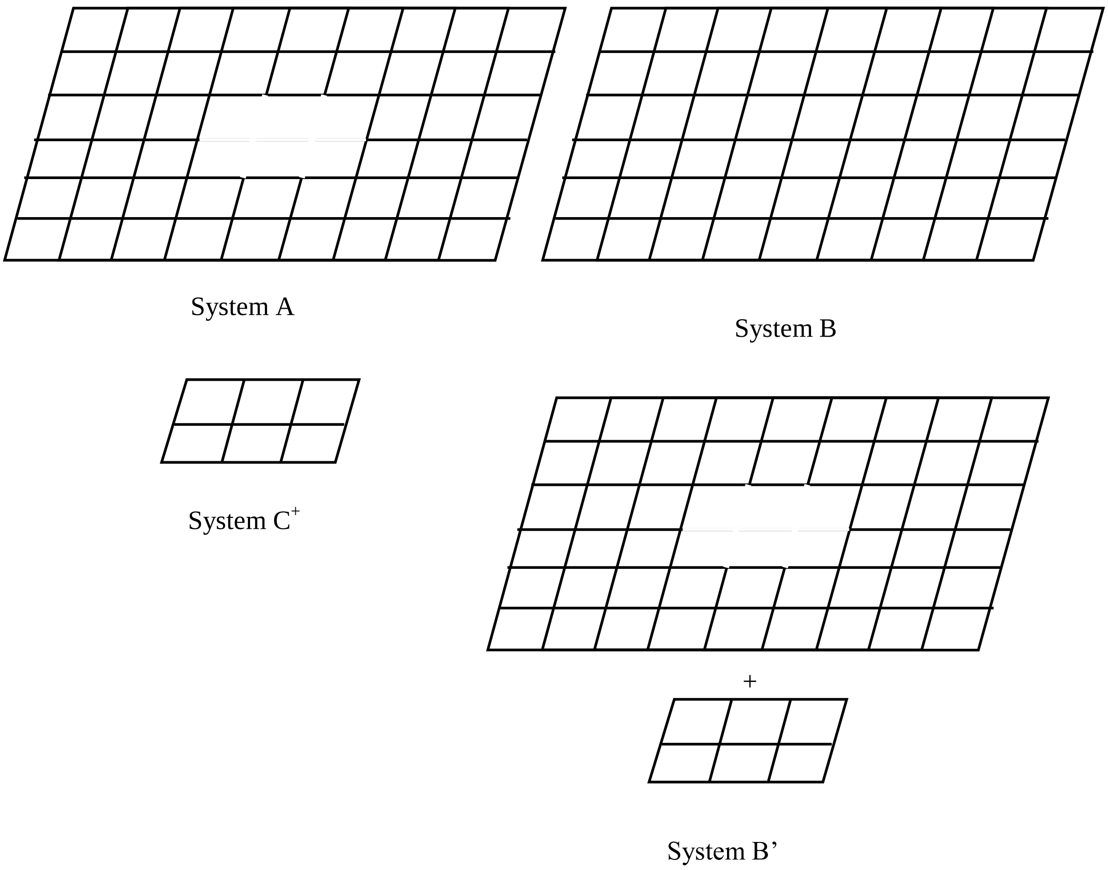

Figure 1

Here, the connection constraints are such that each of the corresponding pairs of r degrees of freedom of A and C^+ vibrate together. The constraint conditions may be written as

$d_{A_i^C} - d_{C_i^C} = 0$, for $i = 1,2..r$,

(1)

Where $d_{A_i^C}, d_{C_i^C}$ are the common degrees of freedom in A and C+.

There is an implication that $r < n$, and $r < m$.

Let $[\chi_A], [\chi_B], [\chi_C]$ be the normalised modal matrices of A, B, C respectively.

Then the *i*th modes of A, B, C are given by the *i*th columns of the corresponding matrices and are denoted by vectors $\{\chi_{A,i}\}, \{\chi_{B,i}\}\{\chi_{C,i}\}$ respectively.

Let $V_A(\bar{f}_A), V_B(\bar{f}_B), V_C(\bar{f}_C)$ etc, be the potential energy of A, B, C+ expressed in terms of assumed displacement vector forms$\{\bar{f}_A\}$, $\{\bar{f}_B\}$, $\{\bar{f}_C\}$ respectively. The displacement forms are obtained from a liner combination of sets of admissible vectors $\{\bar{\phi}_A\}, \{\bar{\phi}_B\}, \{\bar{\phi}_C\}$ which have the same length as the corresponding number of dofs.

i.e. $\bar{f}_{A,j} = \sum_i^n a_i \bar{\phi}_{A,j,i}$, $\bar{f}_B = \sum_i^{n+m-r} a_i \bar{\phi}_{B,j,i}$, $\bar{f}_C = \sum_i^m a_i \bar{\phi}_{C,j,i}$ (2a,b,c)

Similarly the kinetic energy functions of A, B, C^+ are $\psi_A(\bar{f}_A), \psi_B(\bar{f}_B), \psi_C(\bar{f}_C)$.

The energy terms of a negative structure C^- are simply negative of their positive counterparts.

i.e. $V_{C^-}(\bar{f}_C) = -V_C(\bar{f}_C)$; $\psi_{C^-}(\bar{f}_C) = -\psi_C(\bar{f}_C)$ (3a,b)

For discrete systems, the exact frequencies may be readily obtained by minimising the Rayleigh coefficients in terms of an assumed form. For example, for A, an assumed form of displacement $\bar{f}_A$ formed by the admissible functions $[\bar{\phi}_A]$, will give the natural frequencies and modes of A using the Rayleigh-Ritz Method (RRM), if the functions$\{\bar{\phi}_A\}$ are independent, and the number of admissible functions is equal to the number of degrees of freedom.

In terms of the exact modes, the *i*th eigenvalue (the square of the natural frequency) of A is [4]

$$\omega_{A,i}^2 = \frac{V(\chi_A(i))}{\psi(\chi_A(i))} \tag{4a}$$

But from Rayleigh's Principle, the *i*th eigenvalue of A is also given by

$$\omega_{A,i}^2 = \min \frac{V(\bar{f}_A)}{\psi(\bar{f}_A)} \,|< \bar{f}_A, \chi_{A,j} > \text{, for } j = 1,2..i-1 \tag{4b}$$

Now consider the combination of A and C^+, without the connections (see Figure 1), which for convenience may be labelled B′.

$\mathrm{B}' = \mathrm{A}\cup\mathrm{C}^+$

(5)

In B′, it is possible for one of the constituting structures to vibrate in a natural mode while the other remains stationary. Therefore, the eigensolutions of B′ are combination of the eigensolutions of A and C^+.

$$\omega_{B',i}^2 = \min \frac{V(\bar{f}_A)+V(\bar{f}_C)}{\psi(\bar{f}_A)+\psi(\bar{f}_C)} |< \bar{f}_{B'}, \chi_{B',j} > \text{, for } j = 1,2..i-1 \tag{6}$$

But $\mathrm{B} = \mathrm{A}\cup\mathrm{C}^+|$ eq.(1)

(7)

$$\omega_{B,i}^2 = \min \frac{V(\bar{f}_A)+V(\bar{f}_C)}{\psi(\bar{f}_A)+\psi(\bar{f}_C)} |< \bar{f}_{B'}, \chi_{B',j} > \text{, for } j = 1,2..i-1 \text{ and eq.(1)} \tag{8}$$

Here, $\bar{f}_{B'}$denote the union of admissible forms for A and C^+ and , $\chi_{B',j}$ denotes the *j*th mode from the union of the sets of modes of A and C^+.

Since all possible degrees of freedom have been included in the chosen set of admissible forms and the constraint conditions (1) are enforced, eq. (8) gives the exact natural frequencies of B. Thus the natural frequencies of B may be obtained by using the admissible displacement forms for A and C^+ in the RRM.

Now let us consider systems A, B and C as discretised models of continuous systems $\mathbb{A}$, $\mathbb{B}$, and $\mathbb{C}$ with numbers of dofs that are of interest (let us refer to these as "significant degrees of freedom") being *n*, *m* and *n*+*m*-*r* respectively. It is noted here that in order to get accurate modal values for these displacements, additional degrees of freedom may have to be introduced in the models, such that the required natural frequencies and modes of all these systems are correct to the desired degree of accuracy. Therefore, for all practical purposes, the discretised models with the significant degrees of freedom can be used instead of the actual system. The significant dofs correspond to displacements (translations or rotations) at specified locations. The question then arises as to how to relate this to a typical Rayleigh-Ritz analysis of a continuous systems.

Let the actual number of degrees of freedom needed to obtain results with any desired level of convergence be n_B and n_C for systems B and C, where $n_B >> n+m-r$ and $n_C >> m$.

Then a linear combination of admissible functions $\varphi_{A,i}$ for i=1,2.. n_B and $\varphi_{C,j}$ for j=1,2.. n_C will yield independent coordinate values for all of the required number of independent degrees of freedom. This proves that the application of the Rayleigh Ritz Method to an assembly of structures based on the admissible functions for the component structures with the application of relevant connection constraints will yield the natural frequencies and modes of the assembled structure.

The proof above is applicable for any Rayleigh-Ritz model of continuous systems consisting of positive structures. If this argument is valid for combining positive and negative structures, we could then also infer that the natural frequencies of A could be obtained by combining the modes of B and C^- in a Rayleigh-Ritz procedure subject to continuity constraints at the interface/boundary. This would prove to be useful because the natural frequencies and modes of negative structures are identical to those of their positive counterparts because both kinetic energy function and potential energy for the negative structures are equal and opposite to their positive counterparts as given in eq. (3). However, it is not clear whether the Rayleigh's theorem of separation is applicable for negative structures. Currently work is in progress to find a proof that is applicable for assembling negative and positive structures.

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Identification of cracks and temperature in restrained beams using dynamic stiffness matrix

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Abstract

The inverse problem of structural damage identification involves confirmation of damage occurrence, estimation of damage severity and identification of its location. When identifying damage through changes in the modal parameters of the structure, the first two stages are limited by the sensitivity of the equipment used, but having confirmed the existence of damage comes the natural question of where it is necessary to carry out inspections and remedial action.

As is explained in [5], natural frequencies ratio of cracked frame structures ω_i/ω_{io} can be estimated accurately and efficiently using a dynamic stiffness approach. Comparison of these numerical results with natural frequency measurements enables predictions to be made of the most likely places where a detected damage is located. By separating the effects of damage severity, $\tilde{S}$ and location $f_i(x/l)$ it is possible to determine a normalised vector of damage $\delta_i(x/l)$ at different positions through the frame following these basic equations:

$$\delta_i = 1 - \frac{\omega_i}{\omega_{io}} = \tilde{s}.f_i(x/l) \tag{1}$$

$$\delta_i(x/l) = \frac{\delta_i}{\sqrt{\sum_i \delta_i^2}} = \frac{f_i(x/l)}{\sqrt{\sum_i (f_i(x/l))^2}} \tag{2}$$

Each normalised vector has unit magnitude and is a function of damage location but is independent of the severity, having a specific number of elements related to the number of natural frequencies that can be measured.

In order to improve the prediction when other effects additional to the damage occurs, it is paramount to identify and eliminate them. It is known and mentioned in [7] that temperature changes in restricted structures produce additional axial forces which provoke a decrease in the frequency of vibration. These changes give a high degree of uncertainty to the damage prediction because they are of the same order of magnitude as those expected due to the damage. Usually the temperature is treated as noise and is eliminated. However in this work the changes due to temperature are identified and used as known values to assist the localization of the crack in the structure.

The procedure uses the dynamic stiffness matrix with the approximations to the dynamic stability functions proposed by Doyle [2]. The linear form is obtained from the exact transcendental expressions using MacClaurin series to maintain the low order terms. As a result the transcendental dynamic stiffness matrix is replaced by the linear sum of three matrices

which contain separately the terms involving rigidity, inertia and load making it possible to calculate approximate natural frequencies or axial load factors.

The dynamic stiffness matrix has to include the additional force produced by the temperature rise in the restrained beam and there is a consequent diminution in the natural frequencies. When axially loaded beams are analysed, Galef [3] noted a proximity to linear behaviour whereby a straight line relationship is expressed between frequency squared and axial load. The accuracy of this is evaluated in this paper in intervals where the linear behaviour is lost, i.e. when the axial load is almost zero or approaches the critical buckling load in accordance with the results of Bokaian [1], in order to provide a simple prediction of the effect of axial load on the natural frequencies and thus concentrate only on the changes that may be caused by damage and identify the possible location of them.

One of the advantages of using the transcendental dynamic stability functions is that they satisfy the partial differential equation governing the flexural motion of Euler–Bernoulli beams exactly. This form of the dynamic stiffness matrix makes it possible to add into each degree of freedom spring-mass systems that could represent the local flexibility generated by cracks, allowing treatment of problems of multi-cracked structures when one or all of the members carries axial loads. When using these functions in the dynamic stiffness matrix, for example to determine the natural frequencies of a complex beam system, exact results can be obtained with certainty by solving the resultant transcendental eigenproblem using an algorithm devised by [6].

Sample results for a clamped beam using Doyle's approximation are given in Table 1 and the variation of the natural frequency with the location of the spring is shown in Figure 1.

Table 1. Effect of single crack position on the first natural frequency of a clamped beam for various axial loads.

Clamped Beam (k=325kN/rad)							
La (m)	P/Pc						
	0.05	0.1	0.15	0.8	0.85	0.9	0.95
0.01	4.57271	4.51444	4.45383	3.23339	3.05795	2.84575	2.57142
0.05	4.75926	4.69862	4.63554	3.36531	3.18271	2.96184	2.67633
0.10	4.96859	4.90528	4.83942	3.51332	3.32270	3.09212	2.79404
0.20	5.00160	4.93788	4.87158	3.53667	3.34477	3.11266	2.81261
0.25	4.76088	4.70022	4.63712	3.36645	3.18379	2.96285	2.67724
0.49	3.72440	3.67694	3.62758	2.63355	2.49065	2.31782	2.09438
0.50	3.72242	3.67499	3.62565	2.63215	2.48933	2.31658	2.09327
0.51	3.72440	3.67694	3.62758	2.63355	2.49065	2.31782	2.09438
0.75	4.76088	4.70022	4.63712	3.36645	3.18379	2.96285	2.67724
0.80	5.00160	4.93788	4.87158	3.53667	3.34477	3.11266	2.81261
0.90	4.96859	4.90528	4.83942	3.51332	3.32270	3.09212	2.79404
0.95	4.75926	4.69862	4.63554	3.36531	3.18271	2.96184	2.67633
0.99	4.57271	4.51444	4.45383	3.23339	3.05795	2.84575	2.57142

When comparing data, the results indicate a similar pattern between axial load and damage severity comproved with the graph obtained using dynamic stiffness matrix and results presented in [4].

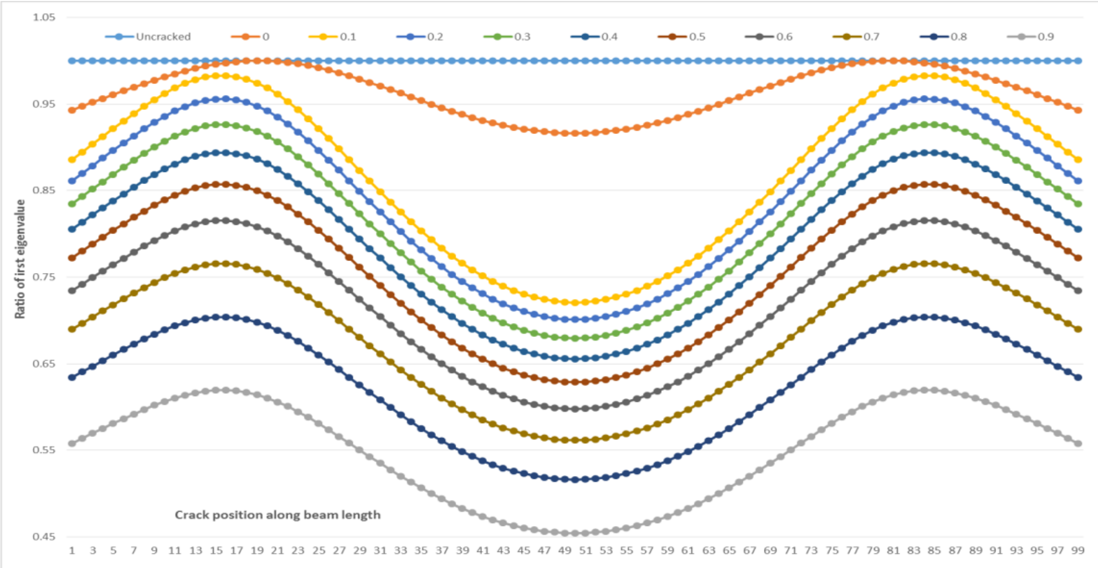

Figure 1. Effect of single crack position on the first natural frequency of a clamped beam for various axial loads (0 to 90 percentage of critical load).

The modified dynamic stiffness matrix is applied to structures composed of beam-column elements, specifically to multi-cracked beams where the cracks are modelled as rotational springs whose rigidity is determined by factors derived from fracture mechanics. A study of the differences in the modal parameters between the exact and the approximated matrices brings a useful perspective on the effectiveness of the latter and its future application to the identification of multiple damage in structures which are also exposed to temperature changes.

The symposium presentation will include experimental results from a planar frame with elements restricted axially and imposed to temperature changes.

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Analysis on Nonlinear Coupled Vibrations in Lateral and Axial Directions of a Post-buckled Beam with Concentrated Masses

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Abstract

Recently, technology of micro electro-mechanical systems (MEMS) has been developed drastically. In such devices, lightweight thin elements such as plates and beams are widely utilized. In practical applications, the boundaries of the plates and beams are often constrained elastically by other elastic structures. Furthermore, concentrated masses might be added at an arbitrary position on the elements or the boundaries of which in-plane or axial inertia would cause drastic change in nonlinear responses. Therefore, in this paper, analytical results are presented on coupled vibrations in lateral and axial directions of a post-buckled beam with concentrated masses. In the analysis, the beam is divided into a few segments. The deflection of the beam is expanded with the mode shape function proposed by the senior author that is expressed with the product of truncated power series and trigonometric functions. Taking the axial displacements, the deflections, slopes, bending moments and shearing forces at the nodes of the segments as unknown variables, nonlinear coupled ordinary differential equations are derived with the Galerkin procedure, which enables the nonlinear analysis considering the axial inertia of concentrated masses and a mass at the end of the beam.

Figure 1 shows the analytical model of the post-buckled clamped beam elastically constrained at an end. We introduce the x and z axes along the axial and lateral directions of the beam, respectively. The origin is taken at the fixed end of the beam. The symbols L and K denote the length of the beam and the spring constant of the axial spring, respectively. The beam is divided into N segments as shown in the figure. The beam has concentrated masses M_1, M_2…, M_N at the nodes of segments. The local coordinate in the n-th segment x_n is introduced which spans from x_n =-1/2 to x_n =1/2. The length, mass density, Young's modulus, area and moment of a cross

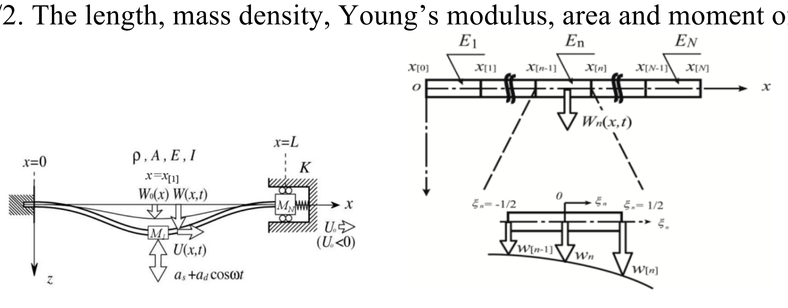

Figure 1. Analytical model of the beam.

section of the n-th segment are expressed by l_n, ρ_n, E_n, A_n and I_n, respectively. The beam is subjected to the static and periodic acceleration $a_s+a_d\cos\Omega t$. The deflection and axial displacement of the beam is expressed as $W(x,t)$ and $U(x,t)$, respectively. The beam is buckled with the initial axial displacement of the axial spring U_0. For sufficiently thin beams, the axial inertia of beam itself, rotational inertia and shearing deformation can be neglected. The non-dimensional governing equation of the vibrations of the beam is expressed as follow, with the Hamilton's principle.

$$\int_{\tau_o}^{\tau_1}\left\{\sum_{n=1}^{N}\left[\int_{-1/2}^{1/2}G_w(w_n)\delta w_n d\xi_n+\left[q_{xn}\delta w_n\right]_{-1/2}^{1/2}-\left[d_n m_{xn}\delta w_{n,\xi_n}\right]_{-1/2}^{1/2}+\left[n_{xn}\delta u_n\right]_{-1/2}^{1/2}\right]\right.$$
$$\left.+\sum_{n=1}^{N-1}\left[\left[\beta_n w_{[n],\tau\tau}-\beta_n p\right]\delta w_{[n]}+\left[\Gamma^2\beta_n u_{[n],\tau\tau}\right]\delta u_{[n]}\right]+\left[\Gamma^2\beta_N u_{[N],\tau\tau}+k\left(u_{[N]}-u_0\right)\right]\delta u_{[N]}\right\}d\tau=0$$

$$G_w(w_n,u_n)=d_n^{-1}\overline{\rho_n}\,\overline{A_n}w_{n,\tau\tau}-d_n n_{xn}w_{n,\xi_n\xi_n}-d_n m_{xn,\xi_n\xi_n}-d_n^{-1}\bar{\rho}_n\bar{A}_n p-\frac{1}{d_n}\delta(\xi-\xi_s)q_s$$

$$n_{xn}=d_n\bar{E}_n\bar{A}_n\left(u_{[n]}-u_{[n-1]}\right)+\frac{1}{2}d_n{}^2\bar{E}_n\bar{A}_n\int_{-1/2}^{1/2}w_{n,\xi_n}{}^2\,d\xi_n \tag{1}$$

In the above equation, d_n is defined as $d_n=L/l_n$, the symbols w_n, u_n, u_0, n_{xn}, s_{xn}, m_{xn} and q_{xn} are non-dimensional deflection, axial displacement, initial axial displacement, axial force, slope, bending moment and shearing force, respectively. Non-dimensional lateral acceleration is denoted by $p=p_s+p_d\cos\omega\tau$, ω and τ are non-dimensional excitation frequency and time, k, β_n and Γ are the non-dimensional axial spring constant, the concentrated mass and the slender ratio of the beam. A vector $\{w_{en}\}$ that consists of nodal variables w_n, s_{xn}, m_{xn} and q_{xn} at the both nodes of the n-th segment is introduced, then the deflection w_n in the n-th segment is expressed with the coordinate function $\{\zeta_n\}$, following the similar manner of the finite element procedure.

$$\begin{gathered}w_n(\xi_n,\tau)=\sum_{j=1}^{8}w_{enj}(\tau)\zeta_{nj}(\xi_n),\{\zeta_n\}=\{\bar{Z}_n\}^T([D_n][Z_n])^{-1},\\ \bar{Z}_{ni}=\sum_{l=1}^{2}\sum_{k=1}^{4}\delta_{i,f(k,l)}\left\{(2\xi_n)^{k-1}\cos(l-1)\pi\left(\xi_n+\frac{1}{2}\right)\right\},f(k,l)=4(l-1)+k\end{gathered} \tag{2}$$

In the above equations, $\{\bar{Z}_n\}$ is a vector composed of the mode shape function Z_{ni} that is the product of truncated power series and trigonometric functions, $[Z_n]$ is a 8×8 matrix consists of Z_{ni} and its first, second and third order derivatives, $[D_n]$ is a 8×8 matrix consists of parameters of the n-th segment. Introducing the global nodal vector $\{\hat{b}\}$ which includes the axial displacement $u_{[n]}$ at the all nodes as well as the nodal vector $\{w_{en}\}$ of the all segments, and applying the Galerkin procedure, Equation (1) is reduced to a set of ordinary differential equations as follows.

$$\begin{gathered}\int_{\tau_0}^{\tau_1}\sum_p\left[\sum_q\hat{B}_{pq}\hat{b}_{q,\tau\tau}+\sum_q\hat{C}_{pq}\hat{b}_q+\sum_q\sum_r\hat{D}_{pqr}\hat{b}_q\hat{b}_r\right.\\ \left.+\sum_q\sum_q\sum_r\hat{E}_{pqrs}\hat{b}_q\hat{b}_r\hat{b}_s-\hat{F}_p-\hat{G}_p(p_s+p_d\cos\omega\tau)\right]\delta\hat{b}_p d\tau=0(p,q,r,s=1,2,3,\cdots,M)\end{gathered} \tag{3}$$

Neglecting the time variant terms, static deflection due to the static lateral acceleration and the axial initial displacements is obtained. Next, the ordinary differential equation is transformed to the equation in terms of the dynamic variable $\tilde{b}_j$ which is measured from the static equilibrium position. Furthermore, the ordinary differential equations are transformed to the standard form in terms of normal coordinates b_i corresponding to the linear natural modes of vibration $\tilde{\zeta}_j$ at the static equilibrium position of the beam as follows.

$$b_{i,\tau\tau} + 2\eta_i\omega_i b_{i,\tau} + \omega_i^2 b_i + \sum_j \sum_k D_{ijk} b_j b_k + \sum_j \sum_k \sum_l E_{ijkl} b_j b_k b_l - p_d G_i \cos\omega\tau = 0 \quad (i,j,k,l = 1,2,3,...,M) \tag{4}$$

In the above equation, modal damping ratio η_i is introduced. Dynamic responses can be calculated with the harmonic balance method and the numerical integration.

Analysis is conducted for a uniform beam. The beam is divided into two segments. Parameters are selected following the previous experiment [1] by the authors as follows.

$$\Gamma = 6.05\times10^{-4}, u_0 = -6600, p_s = 271, k = 7.95\times10^{-3} \tag{5}$$

The deflection is measured at ξ=0.3. Damping ratios for the each mode are η_i=0.008 for the modes ω_i<500 and η_i=0 for the modes ω_i>500. First, the nonlinear frequency response curves are calculated for the sufficiently small concentrated mass $\beta_1=\beta_2=1.0\times10^{-3}$ to compare with the previous analytical results [2] by the authors which neglected the axial inertia force. Figure 2(a) shows the nonlinear response curves calculated by the proposed method (which is indicated by FSA (=finite segment analysis) in the figure) and by the previous results. Although the maximum amplitudes are different in the proposed and previous method, quite good agreement is obtained in the nonlinear characteristics of a softening-and-hardening spring in the response curves in the both results. Effects of a concentrated mass on the nonlinear response curves are investigated comparing the two results of different concentrated mass at the center of the beam, i.e., β_1=0.1, β_2=2.46 and β_1=0.5, β_2=2.46. Figure 2(b) shows the nonlinear response curves under the two conditions. By increasing the concentrated mass at the center, the amplitude of lateral periodic force increases and the natural frequency of the beam decreases, then the amplitude of the fundamental resonance of the lowest mode increases. Figure 2(c) shows the nonlinear response curves under two conditions β_1=0.1, β_2=2.46 and β_1=0.1, β_2=12.3 in which the mass at the end is changed. There is not significant difference in the relatively lower amplitudes. However, the large-amplitude response with the larger end-mass drastically shifts to the lower frequency. This is because the softening-type nonlinearity appears due to the axial inertia of the end-mass. Figure 2(d) compares the previous experimental result [1] and the present analytical result under $\beta_1=1.0\times10^{-3}$, β_2=12.3 of the nonlinear response curves. The experimental result is shown with the thin line and the present analytical result is shown with the thick line. The previous analytical result [2] neglecting the axial inertia is also shown in the figure with the dashed line. It can be found that the analytical result agrees quite well with the experiment by considering the axial inertia of the end-mass in the analysis.

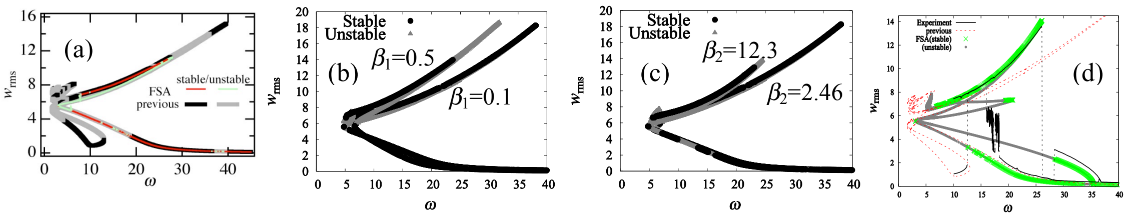

Figure 2. Frequency response curves.

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Connection types of a system consisting of positive and negative beams in free vibration analysis

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Abstract

The existence of natural frequencies and modes of a solid structure on which a negative is attached was discussed in [1]. Another paper gave an example of such structures consisting of continuous systems, namely beams, to explain this using the Dynamic Stiffness Method [2]. The negative beam shares common nodes with the positive beams at its boundary and their internal nodes are connected using penalty springs (Figure 1a). It has been shown that the natural frequencies and modes of the actual structure can be obtained regardless of the internal spring stiffness when the displacements of the positive and negative beams are equal. In this paper, some results for the natural frequencies and modes of the above structure obtained without the internal connection between the positive and negative structures are given. We also present results for another case where the negative beam is connected to the positive beams using large stiffness springs at its boundary (Figure 1b).

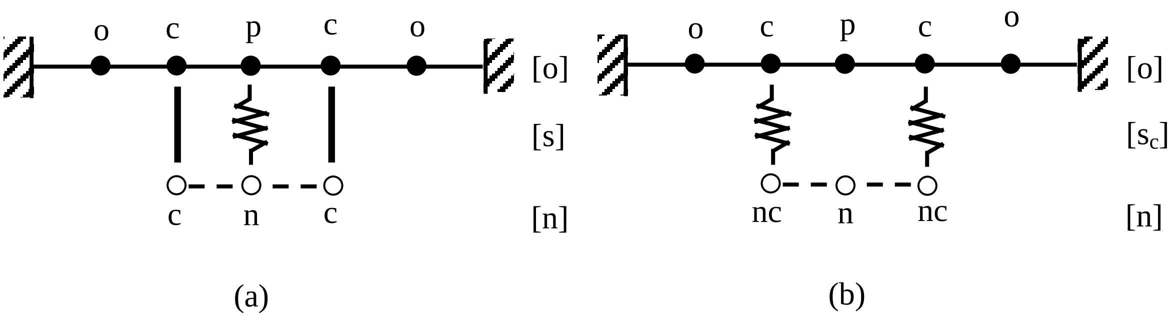

Figure 1. (a) Original structure [o] linked to positive [p] and negative [n] structures attached by springs [s] and sharing common nodes [c], (b) Original structure [o] linked to positive [p] and negative [n] structures attached by springs at boundary only [s_c],

The dynamic stiffness matrix and displacement vectors for the system shown in Figure 1a is given as [2],

$$\begin{pmatrix} \mathbf{K}_{oo} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{oc} \\ \mathbf{0} & \mathbf{k}_{11}+\mathbf{S} & -\mathbf{S} & \mathbf{k}_{1c} \\ \mathbf{0} & -\mathbf{S} & -\mathbf{k}_{11}+\mathbf{S} & -\mathbf{k}_{1c} \\ \mathbf{K}_{oc}^{\mathrm{T}} & \mathbf{k}_{1c}^{\mathrm{T}} & -\mathbf{k}_{1c}^{\mathrm{T}} & \mathbf{K}_{cc} \end{pmatrix} \begin{pmatrix} \mathbf{d}_o \\ \mathbf{d}_p \\ \mathbf{d}_n \\ \mathbf{d}_c \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{1}$$

The subscripts *o*, *p*, *n* and *c* denotes that the internal nodes of original structure, positive and negative beams, and common nodes respectively. The submatrix $\mathbf{S}$ is a diagonal matrix of spring stiffnesses, however, it is possible to obtain the natural frequencies of the original structure without them. The dynamic stiffness equations for the system shown in Figure 1b can be written as follows.

$$\begin{pmatrix} \mathbf{K}_{oo} & \mathbf{0} & \mathbf{0} & \mathbf{K}_{oc} & \mathbf{0} \\ \mathbf{0} & \mathbf{k}_{11} & \mathbf{0} & \mathbf{k}_{1c} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & -\mathbf{k}_{11} & \mathbf{0} & -\mathbf{k}_{1c} \\ \mathbf{K}_{oc}^{\mathrm{T}} & \mathbf{k}_{1c}^{\mathrm{T}} & \mathbf{0} & \mathbf{K}_{cc}+\mathbf{k}_{cc}+\mathbf{S}_{c} & -\mathbf{S}_{c} \\ \mathbf{0} & \mathbf{0} & -\mathbf{k}_{1c}^{\mathrm{T}} & -\mathbf{S}_{c} & -\mathbf{k}_{cc}+\mathbf{S}_{c} \end{pmatrix} \begin{pmatrix} \mathbf{d}_{o} \\ \mathbf{d}_{p} \\ \mathbf{d}_{n} \\ \mathbf{d}_{c} \\ \mathbf{d}_{nc} \end{pmatrix} = \begin{pmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{pmatrix} \tag{2}$$

where the subscript *nc* denotes for the boundary nodes of the negative beam and $\mathbf{S_c}$ is the matrix of connecting springs at the boundary. The matrix $\mathbf{S_c}$ is includes terms for the lateral and rotational springs, which means that the differences of the lateral and rotational displacements between the positive and negative beams will asymptotically vanish if the stiffness approaches infinity.

The natural frequencies are obtained using the Wittrick-Williams (W-W) algorithm. Technically speaking, in order to apply the W-W algorithm the dynamic stiffness matrix with a very low trial frequency, e.g. nearly zero, should not have any negatives along its diagonal after the Gaussian elimination. However, in the present case it may include negative terms even with a very low trial frequency as the negative structure is attached. Therefore, only the change in the number of negative signs is focused. When a trial frequency changes the number of negative signs, the distance of the determinants from the zero line for the previous two trial frequencies are examined to delimit the natural frequencies [3].

The natural frequency parameters of the original structure for the first three modes are presented in Table 1 for four cases where; (1) the positive and negative beams share common nodes and are connected with internal spring connection, (2) the two beams share common nodes only and no internal connection, and (3) the negative beam is connected using the positive penalty springs and (4) the negative penalty springs at boundary only. The each mode has coincident frequencies since the original system is structurally symmetric, i.e. two identical cantilever beams attached on the left and right walls.

Table 1. The natural frequency parameter of the original structure, $\Omega = l\sqrt[4]{\omega^2 m/EI}$, where *l* is the length of the resultant cantilever beam.

Mode	**Ω**			
	(1)	(2)	(3)	(4)
1st	1.8752	1.8752	1.8752	1.8752
	1.8752	1.8752	1.8752	1.8752
2nd	4.6943	4.6943	4.6943	4.6943
	4.6943	4.6943	4.6943	4.6943
3rd	7.8547	7.8547	7.8528	7.8547
	7.8547	7.8547	7.8547	7.8547

As can been seen from Table 1, the natural frequency parameters for the cases (1) and (2) are identical. This proves that the natural frequencies of the original structure can be obtained without the internal connection between the positive and negative beams and the connection only at the boundary of the negative beam is sufficient. The values for first and second modes of the case (3) are the same as the cases (1) and (2), however, for the third mode, one of the frequency parameters is slightly less than the others. This may be caused by a numerical round-off error due to the large stiffness of the spring connecting at the boundary. The case (4) gives the same result as the cases (1) and (2). This implies that the negative penalty parameter gives more stable results for the natural frequencies than the positive penalty parameter does.

In reference [2], it is suggested that the systems shown in Figure 1 may have additional spurious eigenvalues that are given by the Equation (3).

$$|\mathbf{k}_{11}| = 0 \tag{3}$$

They are the eigenvalues of the negative beam when its ends are fixed, as well as the counter-part of positive beam, where the negative beam is attached, with fix ends. By examining the graph of the determinant of Equations (1) and (2), it appears that the determinant has coincident roots at the spurious frequencies, which are the frequencies of fixed beam with the same length as the negative beam for the present case. However, there is no change in the number of negative signs along the diagonal at the spurious frequencies. The reason of that may be considered in following way. Equation (3) is the eigenvalue equation of the fix-ends negative beam as well as the positive counter-part. They are identical in values but in sign. If the sign change occurs for the negative beam the opposite sign change should happen for the positive beam because they have the same natural frequencies. Equations (1) and (2) include both the positive and negative parts, and therefore, if the above sign changes happen at the spurious frequencies the total sign change is effectively zero. This is also observed during the numerical experiment. The second and third rows of Equations (1) and (2) change the sign at the spurious frequencies but the others.

The natural frequencies of the system including the positive and negative structures are obtained using the Dynamic Stiffness Method. The negative structure needs to be connected at its boundary only. This has been shown using the example of connecting beams. The above argument may also be valid for thin plate problems.

Acknowledgement

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Natural frequencies of submerged plates using the Rayleigh-Ritz method and the boundary element method

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Abstract

The purpose of this paper is to present a procedure to solve for the natural frequencies and modes of vibration of submerged structures. The procedure uses the Rayleigh-Ritz method to get the *in vacuo* solution and then uses the modes of vibration to get the influence of the fluid, also known as added mass, in the vibration of submerged structures in an infinite and inviscid fluid. The added mass is calculated using the boundary element method using the procedure described by Antoniadis and Kanarachos [1]. A similar approached but using the finite element method to obtain the *in vacuo* solution was presented by Monterrubio and Krysl in [2].

The results presented here are for a cantilever thin plate of sides a and b along axes x, and y; thickness h in the direction of the z axis; Young's Modulus E; Poisson's ratio ν and density ρ. The Rayleigh-Ritz method is an energy method, consisting in the minimization of the strain energy of bending and twisting $V_{\max}$ and kinetic energy function $T_{\max}$ according to the Kirchhoff plate theory are defined as

$$V_{\max} = \frac{1}{2} D \int_0^a \int_0^b \left[\left(\frac{\partial^2 W}{\partial x^2} + \frac{\partial^2 W}{\partial x^2} \right)^2 - 2(1-\nu) \left(\frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} - \left(\frac{\partial^2 W}{\partial y \partial x} \right)^2 \right) \right] dxdy, \tag{1}$$

$$T_{\max} = \frac{\rho h}{2} \int_0^a \int_0^b W^2 dxdy, \tag{2}$$

where D is the flexural rigidity of the plates $D = Eh^3 / 12(1-\nu^2)$, $W(x,y)$ is the deflection shape of the plate defined as

$$W(x,y) = \sum_{i=1}^{n} \sum_{j=1}^{n} a_{ij} \varphi_i(x) \psi_j(y), \tag{3}$$

where a_{ij} are unknown coefficients while $\varphi_i(x)$ and $\psi_j(y)$ are the sets of admissible functions in the x and y directions defined as

$$\varphi_i(x) = x^i \quad \text{if} \quad i = 1,2 \text{ and } 3 \tag{4a}$$

$$\varphi_i(x) = \cos(i-3)\pi x \quad \text{if} \quad i = 4,5,6,\ldots,n \tag{4b}$$

$$\psi_j(y) = y^j \quad \text{if} \quad j = 1,2 \text{ and } 3 \tag{4c}$$

$$\psi_j(y) = \cos(j-3)\pi y \quad \text{if} \quad j = 4,5,6,\ldots,n, \tag{4d}$$

where n is the number of admissible functions in each direction.

This set of admissible function models an unconstrained plate and artificial translational and rotational springs are used to model constraints. Thus the strain energy of the springs has to be added to the strain energy of plate. The main characteristics of the set of admissible functions presented in Equation [4] are a) a large number of functions can be used in the analysis, which allows to model many constraints to through the use of artificial springs and b) results converge fast with respect to the number of terms even when artificial springs are used to model constraints. The introduction of each constraint using artificial springs leads to the loss of a degree of freedom and an eigenvalue converges towards infinity as the value of the artificial spring increases towards infinity. Thus the last eigenvector and natural frequency can be disregarded from the results.

The minimization of the strain an energy terms give the stiffness matrix $\mathbf{K}$ and mass matrix $\mathbf{M}$ of the plate gives a generalized eigenproblem from which it is possible to solve for the natural frequencies and modes of vibration

$$\mathbf{Ka} - \omega^2 \mathbf{Ma} = \mathbf{0} \tag{5}$$

Once the *in vacuo* solution is obtained the modes can be used to calculate the pressure modes, solving the set of potential (Laplacian) problems as defined in [1]. The potential (Laplace) equation for the inviscid fluid domain is

$$\Delta P = 0, \tag{6}$$

where P is the modal amplitude of the fluid pressure. The structure coupling effect at the common fluid-structure surface is

$$\partial p / \partial n = \omega^2 \rho_{\mathrm{F}} \mathbf{U} \cdot \mathbf{n}, \tag{7}$$

where $\partial p / \partial n$ is the normal derivative of the pressure, ρ_{F} is the density of the fluid, $\mathbf{U}$ is the modal amplitude of the structural displacements and $\mathbf{n}$ is the unit normal.

The solution of the Laplace equations is obtained using the boundary element method with N flat boundary elements E_{i} to discretize the surface of the structure D, together with the point collocation technique as described in the work by Pozrikidis [3]. To compute the pressure modes f the following discretized integral equation is applied at the mid-point of each boundary element, denoted by $\mathbf{x}_j^M$, where $j = 1, \ldots N$:

$$f\left(\mathbf{x}_j^M\right) = -2\sum_{i=1}^{N} \left(\frac{\partial f}{\partial n}\right)_i \int_{E_i} G\left(\mathbf{x}, \mathbf{x}_j^M\right) dS(\mathbf{x}) + 2\sum_{i=1}^{N} f_i \int_{E_i}^{PV} \left[\mathbf{n}(\mathbf{x}) \cdot \nabla G\left(\mathbf{x}, \mathbf{x}_j^M\right)\right] dS(\mathbf{x}), \tag{8}$$

where $\mathbf{x}$ is a vector defining the location of the variable “field point”, $\mathbf{x}_0$ is the fixed location of the singular “point” and G is the free-space Green´s function in three dimension’s for the Laplace equation

$$G(\mathbf{x}, \mathbf{x}_0) = \frac{1}{4\pi r}, \tag{9}$$

where $r = |\mathbf{x} - \mathbf{x}_0|$.

Once the pressure modes are calculated it is possible to compute the terms of the added mass matrix $\mathbf{M}^*$ as follows:

$$\mathbf{M}_{ij}^* = \int_D \rho_{\mathrm{F}} \mathbf{\Psi}_i \cdot \mathbf{f}_j \mathbf{n} dD, \tag{10}$$

Where $\mathbf{\Psi}_i$ are the modes of vibration obtained from the eigenvectors of the dry solution and the set of admissible functions. The natural frequencies of the wet structure are obtained solving the generalized eigenproblem

$$\mathbf{\Omega}^2\mathbf{c} - \omega^2\left(\mathbf{I} + \mathbf{M}^*\right)\mathbf{c} = \mathbf{0}\,, \qquad (11)$$

where $\mathbf{\Omega}$ is a diagonal matrix containing the dry eigenvalues and $\mathbf{I}$ is an identity matrix. It is worth noting that the solution of the eigenvalue problem (11) yields the multipliers of the expansion of the solution for the solid $\mathbf{u}_i = \mathbf{c}_i\mathbf{\Psi}_i$.

Results given below with the present approach were obtained using 30 terms in each of the sets of admissible functions in the x and y directions. Table 1 shows the natural frequencies of a square clamped plate of length and width equal to 10m; height 0.238 m; density 7830 m^3; Young's modulus 206.8 GPa and Poisson's ratio 0.3. The density of the fluid (water) was considered to be 1000 kg/m^3. In general results are close to previous publications by Ergin and Ugurlu [4] and by Fu and Price [5].

Table 1. Natural Frequencies of a cantilever square plate in vacuo

Mode	Present	COMSOL	[4]	[5]
	Hz	Hz	Hz	Hz
1	2.0410	2.0435	2.04	2.06
2	5.0015	4.9768	4.98	5.05
3	12.515	12.498	12.48	12.70
4	15.992	15.909	15.82	16.10
5	18.201	18.088	18.04	18.40

Table 2. Natural Frequencies of a cantilever square plate submerged in water

Mode	Present	COMSOL	[4]	[5]
	Hz	Hz	Hz	Hz
1	1.1359	1.1682	1.17	1.17
2	3.3499	3.3009	3.28	3.22
3	8.1616	7.8416	8.00	8.03
4	11.3996	11.0883	11.07	11.21
5	12.8706	12.5669	12.58	12.55

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Extensions of Layerwise Optimization (LO) Approach for Optimum Lay-up Design of Vibrating Laminated Plates

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Introduction

A layerwise optimization (LO) approach was previously proposed by the author for the lay-up optimization problem of laminated composites [1], and has been applied to many applications [2,3]. This optimization process is independent of the analytical process, where various structural analysis methods, including semi-analytical Ritz method, self-made FEM and commercial FEM, are accommodated. Generally speaking in the lay-up design problems, it is typically seen that they cause rapid increase in computation time due to searching optimum solutions in the multi-dimensional space, when design variables are taken to be the fiber orientation angles directly in all layers. The LO makes it possible, however, that this multi-dimensional optimization problem can be reduced into only a finite times repetition of one-dimensional search. But at the same time, it has disadvantages that LO algorithm is occasionally trapped in local solutions, and the improvement of the object functions is quite slow when the fiber orientation angles are searched in inner layers.

Layerwise optimization for vibration of laminated composite plates

Free vibration of symmetrically laminated plates is governed in the classical theory by

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}+4D_{16}\frac{\partial^4 w}{\partial x^3\partial y}+4D_{26}\frac{\partial^4 w}{\partial x\partial y^3}-\rho\omega^2\frac{\partial^2 w}{\partial t^2}=0 \qquad (1)$$

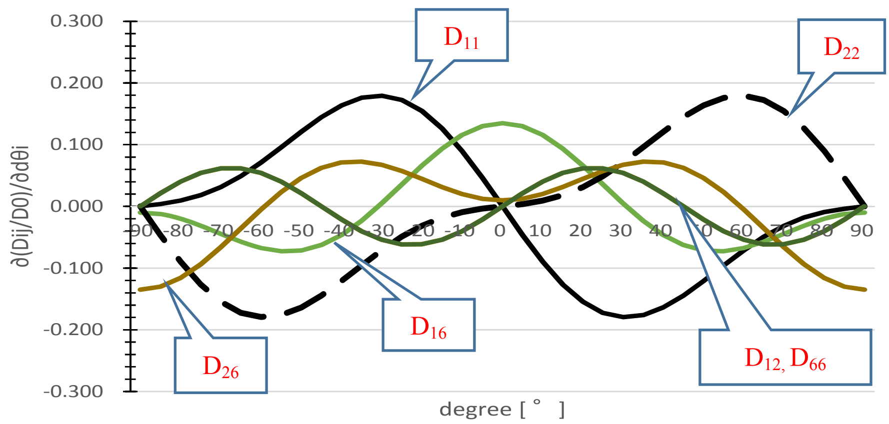
where D_{ij} (i,j=1,2,6) are the bending stiffnesses of the symmetric laminate defined by $D_{ij}=\left(2/3\right)\sum_{k=1}^{N}\bar{Q}_{ij}^{(k)}\left(z_k^3-z_{k-1}^3\right)$. The $\bar{Q}_{ij}^{(k)}$ are elastic constants in the *k*-th layer obtained from the orthotropic material constants and fiber orientation angles in the layers. Solving Eq.(1) is reduced to an eigenvalue problem, where the natural frequency ω is written as a parameter $\Omega=\omega a^2(\rho h/D_0)^{1/2}$ with a reference stiffness $D_0=E_t h^3/12(1-\nu_{12}\nu_{21})$. It is easily recognizable from Eq.(1) that the values of natural frequency ω are significantly dependent on the bending stiffness D_{ij} that are proportional to the cube of thickness coordinate z.

In the present optimization, the frequency parameter Ω_1 for the fundamental mode is used (but not limited to) as the object function and is maximized. The design variables are taken to be a set of fiber orientation angles in the layers of the upper (lower) half of the cross-section $[\theta_1/\theta_2/\ldots/\theta_N]_S$, where θ_k is the fiber orientation angle in the k-th layer (k=1:outermost, $k=N$: innermost). The LO attempts to avoid the computational problem by making use of physical observation, and the next assumption in optimization is proposed in the algorithm:

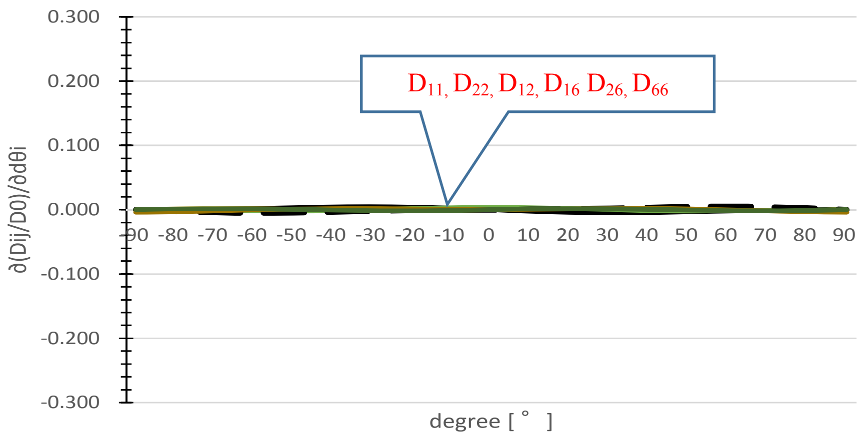
The optimum stacking sequence $[\theta_1/\ \theta_2/\ldots/\ \theta_N]_{S,opt}$ *for the maximum natural frequency of laminated plates can be determined sequentially in the order from the outermost to the innermost layer.*

Table 1 Layerwise optimization for symmetric 8-layer square plate (CSFF, $\Delta\theta$=5°)

	$[\theta_1/\theta_2/\theta_3/\theta_4]$s	Ω_1		$[\theta_1/\theta_2/\theta_3/\theta_4]$s	Ω_1
1st iteration cycle solutions			2 nd iteration cycle solutions		
Step 0	[*/*/*/*]s	–	Step 0	[10/−40/30/25]s	16.04
Step 1	[10/*/*/*]s	11.98	Step 1	[20/−40/30/25]s	16.36
Step 2	[10/− 40/*/*]s	14.75	Step 2	[20/−45/30/25]s	16.38
Step 3	[10/−40/30/*]s	15.89	Step 3	[20/−45/20/25]s	16.40
Step 4	[10/−40/30/25]s	16.04	Step 4	[20/−45/20/25]s	16.40
			3rd iteration cycle (same as 2nd)		



(a) Variation of sensitivity, $\partial(D_{ij}/D_0)/\partial\theta_1$, versus θ_1 (outermost layer)



(b) Variation of sensitivity, $\partial(D_{ij}/D_0)/\partial\theta_4$, versus θ_4 (innermost)

Fig.1 Sensitivity variations versus fiber orientation angles in the 1st iteration cycle (Table 1).

Drawback of Simple LO (SLO)

Here a traditionally used LO is named as Simple LO (SLO). It has been successfully used in many papers. One example is shown in Table.1, where the fundamental frequency is maximized with $[\theta_1/\theta_2/\theta_3/\theta_4]_S$ for an eight-layer square plate with Clamp-Simple support-Free-Free edges. Figure1 presents variations of sensitivity (first derivative) of Dij with respect to (a) outermost layer θ_1 and (b) innermost layer θ_4. It is clearly seen that the innermost layer does not affect the stiffnesses in the optimization process.

Introduction of Layer Dominance Ratio (LDR)

As seen in previous example, inner layers (most significantly, innermost layer) contribute little to improvement of the objective function, and therefore small angle increment (e.g., $\Delta\theta=5°$) in inner layers is meaningless in the optimization. Since the bending stiffness D_{ij} is proportional to the cube of thickness, and the effect of the angle can be considered to be proportional to the value of $L(\mathrm{i})=(2/h)^3(z_\mathrm{i}^3-z_{\mathrm{i}-1}^3)$ (i: layer number), which may be called Layer Dominance Ratio (LDR). In the case of symmetric 8 layer plate, these values are $L(1)=0.578$, $L(2)=0.297$, $L(3)=0.109$ and $L(4)=0.016$. The sum of all $L(\mathrm{i})$ is one. For example, the outermost layer of a symmetric 8-layer plate is more influential than the innermost layer by $L(1)/L(4)=0.578/0.016=37$ times in the bending stiffness. In an extension of the LO process, LDR is used to reduce the number of frequency calculations. When $\Delta\theta_1=5°$ is assumed, $\Delta\theta_2=5°\times L(1)/L(2)=9.73°$ or approximately 10°, and likewise for $\Delta\theta_3$ and $\Delta\theta_4$, may be used to give results with almost equivalent accuracy, but with significantly less number of calculations than SLO.

Use of LDR in random search

The concept of LDR can be used in other methods of lay-up design problems, and one example is to use LDR to decide which layer should be chosen in the random optimization. In the numerical experiment for a CSFF square plate, Layerwise Random Search (LRS), that is a random search with consideration of LDR, gave almost the same optimum value shown in Table 1 after only 25 iterations, while purely random search took much longer time.

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Energy localization in carbon nanotubes

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Abstract
In this paper, the energy localization phenomena in low-frequency nonlinear oscillations of single-walled carbon nanotubes (SWNTs) are analysed. The SWNTs dynamics is studied in the framework of the Sanders-Koiter shell theory. Simply supported and free boundary conditions are considered. The effect of the aspect ratio on the analytical and numerical values of the localization threshold is investigated in the nonlinear formulation.

1. Introduction

Carbon nanotubes (CNTs) are used as ultrahigh frequency nano-mechanical resonators in a large number of nano-electro-mechanical devices such as sensors, oscillators, charge detectors and field emission devices. The reduction of the size and the increment of the stiffness of a resonator increase its resonant frequencies and reduce its energy consumption, improving therefore its sensitivity.
The stationary or nonstationary dynamics of CNTs can be treated in terms of linear or nonlinear normal modes; in the presence of non-stationary resonance, one assists to energy transfer phenomena and formation of wave packets, having a time evolution strongly related to the spectral properties. In the nonlinear systems, the wave dispersive spreading can be compensated by the nonlinearity. As a result, a soliton mechanism of energy transfer in the quasi-one-dimensional nonlinear lattices arises.
In the present paper, the energy exchange and transition to energy capture in some part of the CNT is analysed and explained within the Limiting Phase Trajectory theory [1]. Two different approaches are used: 1) a numerical model based on the Sanders-Koiter shell theory, solved semi-analytically through a double mixed series expansion for the displacement fields; 2) an analytical model based on a reduced form of the shell theory assuming small circumferential and tangential shear deformations.

Theory

The numerical approach is based on the Sanders-Koiter shell theory, the strain and kinetic energies are written as [2]:

$$\tilde{E} = \frac{1}{2}\frac{1}{(1-\nu^2)}\left[\int_0^1\int_0^{2\pi}\left(\tilde{\varepsilon}_{x,0}^2 + \tilde{\varepsilon}_{\theta,0}^2 + 2\nu\tilde{\varepsilon}_{x,0}\tilde{\varepsilon}_{\theta,0} + \frac{(1-\nu)}{2}\tilde{\gamma}_{x\theta,0}^2\right) d\eta d\theta + \frac{\beta^2}{12}\int_0^1\int_0^{2\pi}\left(\tilde{k}_x^2 + \tilde{k}_\theta^2 + 2\nu\tilde{k}_x\tilde{k}_\theta + \frac{(1-\nu)}{2}\tilde{k}_{x\theta}^2\right) d\eta d\theta\right] \quad 1)$$

$$\tilde{T} = \frac{1}{2}\gamma \int_0^1 \int_0^{2\pi} (\tilde{u}'^2 + \tilde{v}'^2 + \tilde{w}'^2)\, d\eta\, d\theta \qquad\qquad \gamma = \rho R^2 \omega_0^2 / E \tag{2}$$

The displacement fields are expanded as follows:

$$\tilde{u}(\eta,\theta,\tau) = \sum_{j=1}^{N_u}\sum_{n=1}^{N} \tilde{U}^{(j,n)}(\eta,\theta)\tilde{f}_{u,j,n}(\tau) \quad \tilde{v}(\eta,\theta,\tau) = \sum_{j=1}^{N_v}\sum_{n=1}^{N} \tilde{V}^{(j,n)}(\eta,\theta)\tilde{f}_{v,j,n}(\tau) \quad \tilde{w}(\eta,\theta,\tau) = \sum_{j=1}^{N_w}\sum_{n=1}^{N} \tilde{W}^{(j,n)}(\eta,\theta)\tilde{f}_{w,j,n}(\tau) \tag{3}$$

where the approximate eigenfunctions are obtained through the Rayleigh-Ritz procedure explained in Ref. [2]. The resulting dynamical system is obtained by means of the Lagrange equations which are solved numerically.

It is to note that the present model does not include additional terms in partial differential equations (e.g., non-local moment, Eringen's relation) which allow to consider the "size effects". The reason is that, for the present analysis, focused on low-frequency and long SWNTs, these effects are marginal. Comparisons with Molecular Dynamics simulations confirm that our assumptions are acceptable [3].

An alternative approach is based on a reduced form of the Sanders-Koiter linear elastic shell theory developed in [2] and extended to the nonlinear field. Since low-frequency vibrations of SWNTs are considered in this work, then the elastic strain energy is predominantly due to bending, torsion and longitudinal tensions, and therefore we can neglect the circumferential and tangential shear strains of the middle surface. Due to these assumptions, the longitudinal and circumferential displacements can be expressed via the radial one. Details are omitted for the sake of brevity.

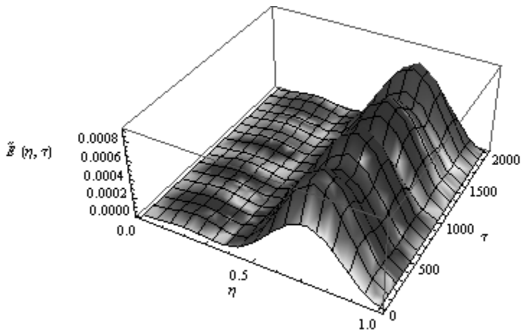

Figure 1. Energy localization: simply supported

Numerical Results

In Figure 1, the total energy distribution over the CNT surface is represented (simply supported edges). When the total energy of vibration is sufficiently high, then the combination of the two modes (1,2) and (2,2) results in a strong localization of the total energy distribution. This is a nonlinear phenomenon as the localization disappears when the vibration energy is low enough (or the system is linearized).

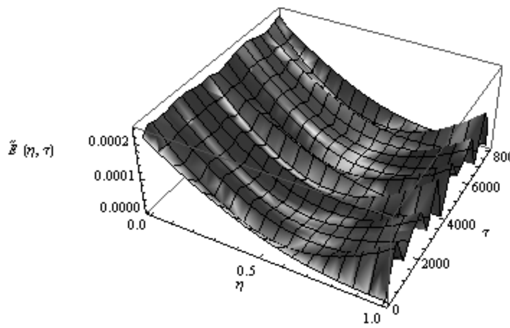

Figure 2. Energy localization: free-free

The same phenomenon appears in the case of a free-free SWNT (modes (0,2) and (1,2)), Figure 2. Also in this case the localization takes place when the vibration energy is sufficiently high.

Figure 3 clarifies that an energy threshold exists for the onset of localization. Different energy levels are needed for simply supported or free-free SWNTs, the behaviour is similar when the aspect ratio is varied, i.e., an asymptotic energy level is found for long SWNTs.

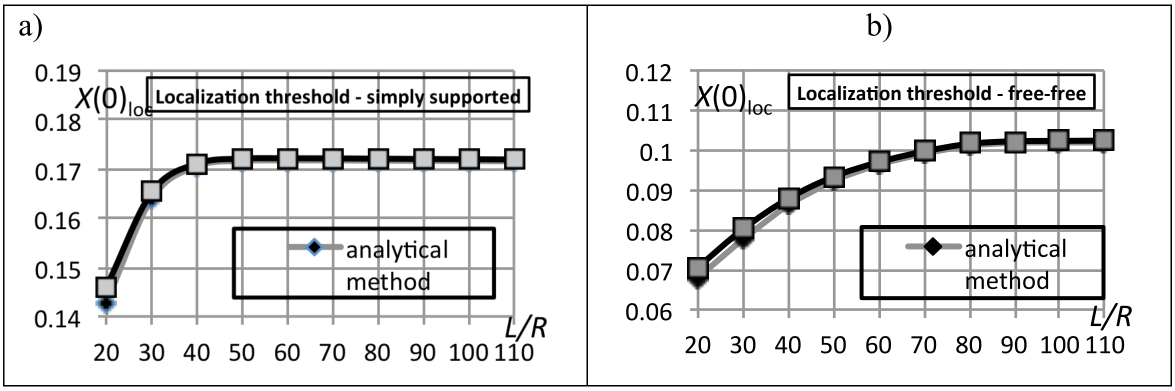

Figure 3. Localization threshold: a) simply support; b) free-free boundary conditions

Conclusions

In this paper, the low-frequency oscillations and energy localization of SWNTs are analysed within the framework of the Sanders-Koiter shell theory. The circumferential flexure modes are considered. Simply supported and free boundary conditions are studied. Two different approaches are compared, based on numerical and analytical models. For the free boundary conditions, the energy localization threshold value at the horizontal asymptote is lower than the corresponding value for the simply supported boundary conditions, since in this particular case the uniform vibrational mode with zero longitudinal half-waves loses its stability at a relatively low energy level.

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The Free Vibration Analysis of Piezoelectric Sandwich Plates with Applications for Energy Harvesting.

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Introduction. The present study is concerned with a general variational method around the non-uniform rational B-spline (NURBS) function for modeling arbitrarily shaped piezoelectric sandwich plates. The formulation is based on the first order shear deformable plate theory according to which the in-plane displacement components vary linearly over the thickness. The nonlinear variation of the electric potential through the piezoelectric medium is accommodated by a layer-wise scheme. The non-uniform rational B-spline (NURBS) functions are used to define the geometry as well as the unknown field functions. The coupled matrix equations of motion are obtained for piezoelectric plates used as sensors and actuators. Subsequently, these are: applied to study the free vibration of circular and elliptic plates and extended further with the help of the normal mode summation method to investigate piezoelectric energy harvesters. Results for the energy harvesting are not reported in this paper, but will be presented in the symposium.

Geometrical representation in NURBS. The NURBS curve using n control points is written as $C(\xi) = \Sigma R_{i,k}(\xi)\ p_i$, where $R_{i,k}(\xi) = N_{i,k}(\xi)\, w_i / \Sigma N_{i,k}(\xi)\, w_i$ and the summation is on $i = 1,2,3,....,n$ (Piegel and Tiller, 1997). The remaining parameters are: $p_i =$ a vector of the control points; $w_i =$ the weights; $k =$ the order; and $N_{i,k} =$ the components of B-Spline basis function of ξ and can be evaluated recursively. Similarly, a surface patch can be generated through the tensor product of the univariate NURBS curves in ξ and η directions and written by $S(\xi,\eta) = \Sigma\Sigma R_{ij}(\xi,\eta)\, p_{ij}$, where $p_{ij} =$ defines the control net points. Summations here are from 1 to n for i and 1 to m for j respectively. The very basic unit of the plate structure is taken to be a sizeable quadrilateral patch bounded by four curved edges as shown in the figure below. The edges are defined by either ξ or η of the natural coordinates. After obtaining the control points of the four outer edges, the control net of the patch is developed by interpolating the intermediate points between a pair of edges.

Equations for the first order shear deformable piezoelectric plate. The response field functions for the plate vibration are defined by NURBS patch as well. Thickness h of the plate is assumed to be constant and displacement components at an arbitrary point varying as $u' = u + z\beta_1$, $v' = v + z\beta_2$ and $w' = w$. Here, u, v and w are displacement components along the x, y, and z axes on the mid-plane of the plate. Similarly, β_1 and β_2 are the components of the rotation of the normal to the plate in the x and y directions respectively. The NURBS surface patches are used also to represent individually the mechanical displacement components of the vector $\{\Delta\}^T = \{u \quad v \quad w \quad \beta_1 \quad \beta_2\}$. The number of unknowns for a patch depends on the control coefficients chosen along ξ and η directions. For piezoelectric plate, there is a potential function ϕ which varies nonlinearly in the thickness direction. This nonlinear characteristic is efficiently dealt with by a layer-wise approach in which the material along the thickness direction is discretized into sub-layers and the potential function is assumed to vary linearly in each sub-layer. The Hamilton's variational principle is used to obtain the following equation (Allik and Hughes, 1970).

$$[M]\{\ddot{\Gamma}(t)\} + [C]\{\dot{\Gamma}(t)\} + [K_m]\{\Gamma(t)\} + [K_{me}]\{\varphi(t)\} = \{F(t)\} \quad \text{and}$$
$$[K_{em}]\{\Gamma(t)\} + [K_e]\{\varphi(t)\} = \{Q(t)$$

The right hand side contains both mechanical and electrical excitations and can be set to zero for the free vibration analysis. It is performed on the elliptic sandwich plates under clamped and simply supported boundary conditions given by $u = v = w = \beta_1 = \beta_2 = 0$ and $u = v = w = 0$ for the clamped simply supported boundary conditions. In both cases, the potential function ϕ on the exposed piezoelectric layers is assumed to be grounded.

Free vibration of circular and elliptic sandwich plates. The plate is composed of a steel layer sandwiched by two piezoelectric PZT4 layers. Thicknesses of the top and bottom layers are denoted by h_1 and h_3 respectively and that of the host steel layer in the middle by h_2. The geometry of the piezoelectric sandwich plate is varied from a circle to an ellipse by changing the a/b ratio. A 3×3 NURBS patch, each containing 36 control points, is used in the plate models. Values of the first six natural frequencies for the clamped circular plate, with $\bar{h} = (h_1 + h_3)/h_2 = 0.2$, are presented in Table 1, which also contains results from a study reported in the literature by Wang et al. (2001). Results from the two studies are in very good agreement, showing the maximum percentage difference of 0.75 at the sixth mode. Some new results are also reported in Table 1for the elliptic plate with a/b = 1.5, 2 and 3. The simply supported circular (a/b = 1) and elliptic (a/b = 2) plates are also analyzed for the thickness ratios 1/10, 1/8, and 1/5. In this case also, the present results are in close agreement with those by Wang et al. (2001).

Closing remarks. The main advantage of the present technique is that it enables one to model complex geometric shapes accurately with less number of degrees of freedom. The numerical simulation results are successfully corroborated by comparing with the exact closed form solution for the free vibration analysis of clamped and simply supported circular plates (Wang et al., 2001). Some new results on the natural frequencies of piezoelectric elliptic sandwich plates are generated and reported. The authors of this paper also applied the NURBS based formulation to investigate the piezoelectric power harvesting devices, (Erturk and Inman, 2011). They considered cantilevered rectangular and curved edged piezoelectric sandwich plates in power harvesting. The influence of load resistance on the power output and free end displacements will be presented.

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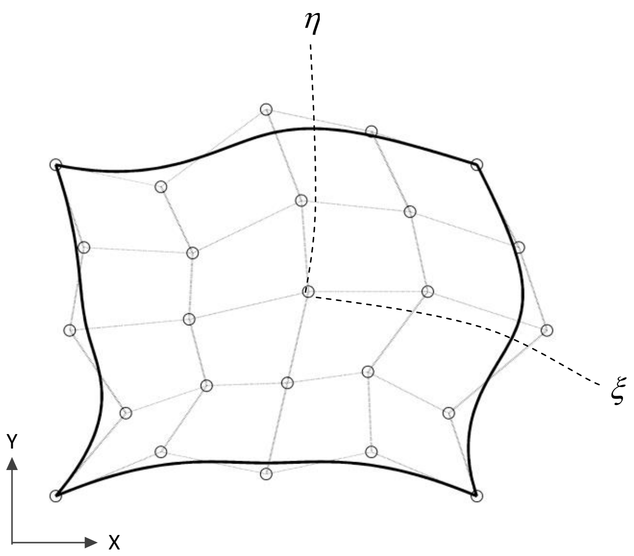

Figure. Quadrilateral middle surface of plate

Table 1. Natural Frequencies of Clamped Sandwich Plate, $\bar{h} = 0.2$

Mode	$a/b=1$			$a/b=1.5$	$a/b=2$	$a/b=3$
	Present	Wang (2001)	Difference (%)			
1	902.51	902.47	0.004	680.06	613.40	567.26
2	1872.38	1878.17	0.309	1121.53	877.60	710.07
3	1873.55	-	-	1645.34	1240.35	891.83
4	3063.13	3081.08	0.586	1747.05	1562.46	1113.42
5	3066.32	-	-	2238.11	1704.31	1378.70
6	3487.05	3513.43	0.757	2550.14	1951.76	1495.37

Table 2. Natural Frequencies of Simply Supported Sandwich Plate,

Mode	$\bar{h}=0.2$, $a/b=1$			$a/b=2$		
	Present	Wang (2001)	Difference (%)	$\bar{h}=0.1$	$\bar{h}=0.125$	$\bar{h}=0.2$
1	449.67	462.33	2.185	290.02	292.20	299.47
2	1241.66	-	-	511.86	515.63	528.22
3	1246.26	1303.26	4.574	827.41	833.44	853.57
4	2281.06	-	-	1006.27	1013.50	1037.67
5	2292.82	2402.29	4.774	1240.26	1249.21	1279.13
6	2643.55	2787.56	5.448	1353.29	1362.90	1395.08

On the eigensolution of elastically connected columns

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Introduction

Consideration is given to determining the eigensolution of a class of structures comprising n suitably related parallel columns that are connected to each other, and possibly also to foundations, by uniformly distributed Winkler interfaces of unequal stiffness. For conciseness, the body of the work is written in the context of elastic critical buckling. However, since the duality of the eigenproblem posed by buckling and vibration is well known, the extension to vibration is straightforward.

Initially the coupled fourth order differential equations that define the system are developed from first principles and arranged in the form of a generalized symmetric linear eigenvalue problem. Exact solution of these equations leads to n, uncoupled substitute systems, each of which yields an infinite number of critical buckling loads that, when arranged in ascending order, comprise the complete spectrum of critical buckling loads of the original problem. Thus, if only the fundamental critical buckling load is required, then only one substitute system needs to be solved.

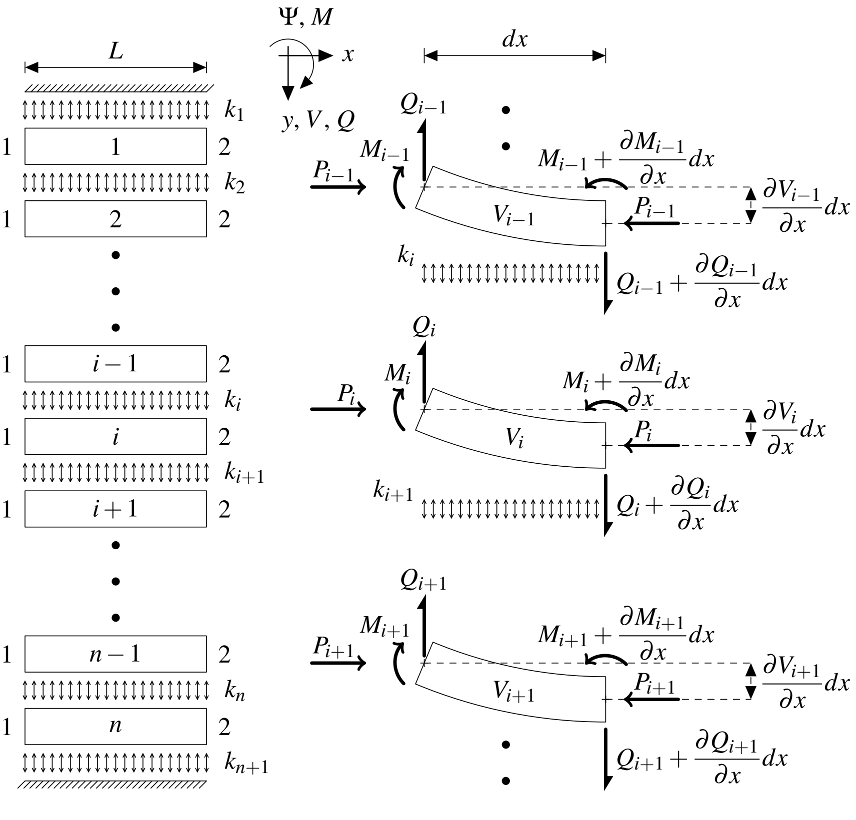
Each substitute system is relatively simple and describes the buckling of a single unified member, but supported on a Winkler foundation of different magnitude in each case. However, the exact solutions required from each substitute system necessitate the closed form solution of a transcendental eigenvalue problem. This is achieved in the present case by utilizing an exact elastic stiffness matrix in conjunction with the Wittrick-Williams algorithm, which guarantees that any desired critical buckling load can be calculated to any desired accuracy with the certain knowledge that none have been missed. The corresponding modes of vibration are then recovered by back substitution for each substitute system and subsequently related back to the individual members of the original structure. This approach also enables some of the powerful features of the stiffness method to be utilized to model more complex structures. A simple example is given to clarify the approach.

Theory

Figure 1 defines the structural configuration and the positive directions of the member forces and displacements, from which the equations of vertical and moment equilibrium for typical member i can be deduced straightforwardly. These are presented with the appropriate constitutive relationship in Equations (1)-(2)

$$-Q_i+(Q_i+\frac{dQ_i}{dx}dx)+k_i dxV_{i-1}-(k_i+k_{i-1})dxV_i+k_{i+1}dxV_{i+1}=0 \tag{1}$$

$$Q_i dx+M_i-(M_i+\frac{dM_i}{dx}dx)+P_i\frac{dV_i}{dx}dx=0 \qquad M_i=-E_iI_i\frac{d^2V_i}{dx^2} \tag{2}$$



(a) (b)

Figure 1: (a) General structure orientation. k_i is a typical Winkler stiffness per unit length. (b) Positive member forces and displacements relating to an elemental length of typical member i.

Eliminating Q_i and M_i and introducing the non-dimensional length parameter $\xi = x/L$ yields

$$-\kappa_i[D^4 + p_i{}^2 D^2]V_i + k_i V_{i-1} - (k_i + k_{i+1})V_i + k_{i+1}V_{i+1} = 0 \tag{3}$$

where

$$D = \frac{d}{d\xi}, \qquad \kappa_i = \frac{E_i I_i}{L^4}, \qquad p_i{}^2 = \frac{P_i L^2}{E_i I_i} \tag{4}$$

The current approach now requires that $p_i{}^2$ is a constant for each member. i.e. that

$$p_i{}^2 = p^2 \tag{5}$$

Equation (3) can then be written for a general member as

$$-k_i V_{i-1} + (k_i + k_{i+1})V_i - k_{i+1}V_{i+1} - \kappa_i \lambda V_i = 0 \tag{6}$$

where

$$\lambda = -[D^4 + p^2 D^2] \tag{7}$$

Hence the corresponding equations for the first ($i = 1$) and last ($i = n$) members are, respectively

$$(k_1 + k_2)V_1 - k_2 V_2 - \kappa_1 \lambda V_1 = 0 \tag{8}$$

and

$$-k_n V_{n-1} + (k_n + k_{n+1})V_n - \kappa_n \lambda V_n = 0 \tag{9}$$

A complete set of equations for an n level system can now be assembled from Equations (6), (8) and (9)

$$\left\{\begin{bmatrix} k_1+k_2 & -k_2 & & & \\ & \ddots & & & \\ & -k_i & k_i+k_{i+1} & -k_{i+1} & \\ & & & \ddots & \\ & & & -k_n & k_n+k_{n+1} \end{bmatrix} - \lambda \begin{bmatrix} \kappa_1 & & & & \\ & \ddots & & & \\ & & \kappa_i & & \\ & & & \ddots & \\ & & & & \kappa_n \end{bmatrix}\right\} \begin{bmatrix} V_1 \\ \vdots \\ V_i \\ \vdots \\ V_n \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{bmatrix} \tag{10}$$

where zeros have been omitted for clarity. Equation (10) can therefore be written for any appropriate formulation as

$$(\mathbf{k} - \lambda\,\boldsymbol{\kappa})\mathbf{V} = 0 \tag{11}$$

The form of Equation (11) is that of a generalized symmetric linear eigenvalue problem, for which a number of standard routines are available for calculating the eigenvalues, λ, and corresponding eigenvectors, $\mathbf{V}$.

Substitute systems

The n values of λ that satisfy the linear eigenvalue problem define a family of second order differential operators that satisfy the original problem and which are given by Equation (7) as

$$D^4 + p^2D^2 = -\lambda_i \qquad i = 1,2,\ldots,n \tag{12}$$

Equation (12) can be assigned a physical context by noting that it is a property of such differential operators that they can be written as

$$[D^4 + p^2D^2]V = -\lambda_i V \qquad i = 1,2,\ldots,n \tag{13}$$

and hence that

$$[D^4 + p^2D^2 + \lambda_i]V = 0 \qquad i = 1,2,\ldots,n \tag{14}$$

and V is a typical lateral displacement.

Each of these equations now describe the elastic critical buckling of a single unified member, but supported on a Winkler foundation of different magnitude in each case. Equation (14) therefore represent n substitute systems, each of which yields an infinite number of critical buckling loads that, when arranged in ascending order, comprise the complete spectrum of critical buckling loads of the original problem. It therefore follows that when only the lowest critical buckling load is required, it is only necessary to solve the substitute system that contains the lowest linear eigenvalue obtained from Equations (10) and (11).

In the current context, each substitute system is solved by transforming to a stiffness formulation in conjunction with the Wittrick-Williams algorithm, the boundary conditions being the single, identical set imposed on each of the original members.

Example

The simple illustrative problem selected can be envisaged by considering Figure 1(a) as comprising five simply supported members ($n = 5$) with $k_i = \kappa_i = k$ ($i = 1,2,\ldots,5$) and $k_6 = 0$. The five linear eigenvalues and their corresponding eigenvectors that then stem from Equations (10) and (11) are given in Table 1.

Table 1: Linear eigenvalues and their corresponding eigenvectors.

		$V_{i,j}$				
i	λ_i	$j=1$	$j=2$	$j=3$	$j=4$	$j=5$
1	0.0810	0.2817	0.5406	0.7557	0.9096	0.9898
2	0.6903	0.7557	0.9898	0.5406	−0.2817	−0.9096
3	1.715	0.9898	0.2817	−0.9096	−0.5406	0.7557
4	2.831	0.9096	−0.7557	−0.2817	0.9898	−0.5406
5	3.683	0.5406	−0.9096	0.9898	−0.7557	0.2817

The first three of a possible infinite number of critical buckling load parameters, p^2, is then given in Table 2. Hence, the critical buckling loads in each member can be deduced from Equations (4) - (5).

Table 2: The first three critical buckling load parameters for each linear eigenvalue. The modal numbers are given in brackets.

	p^2 for each λ_i				
m	$i=1$	$i=2$	$i=3$	$i=4$	$i=5$
1	9.8778 (1)	9.9395 (2)	10.043 (3)	10.156 (4)	10.243 (5)
2	39.481 (6)	39.496 (7)	39.522 (8)	39.550 (9)	39.572 (10)
3	88.827 (11)	88.834 (12)	88.846 (13)	88.858 (14)	88.868 (15)

Mode shape recovery

The buckling mode shape of the original members can be recovered by multiplying the substitute system mode shape by the appropriate element of the appropriate linear eigenvector. Now it is clear that the substitute system mode shapes for a simply supported member are defined by

$$V = A\sin(m\pi\xi) \qquad m = 1, 2, \ldots, \infty \tag{15}$$

where A is an arbitrary constant.

As an example, consider the eigenvector corresponding to the fourteenth critical load of the original structure. It can be seen from Table 2 that this corresponds to $p^2 = 88.858$, with $i = 4$ and $m = 3$. Thus the eigenvector defining the mode shapes of the five original members, top to bottom, is

$$\widetilde{\mathbf{V}} = A\mathbf{V}_{4,j}\sin(3\pi\xi) \qquad j = 1, 2, \ldots, 5 \qquad \text{or} \qquad \widetilde{\mathbf{V}} = A\begin{bmatrix} 0.9096 \\ -0.7557 \\ -0.2817 \\ 0.9898 \\ -0.5406 \end{bmatrix}\sin(3\pi\xi) \tag{16}$$

Vibrations of Cosserat Curved Rods

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Abstract

The static equilibrium, stability and dynamics of elastic curved rods subjected to external loads are a classical problem in the mechanics. A history of the problem and survey of development up to 1993 was given in [1]. Later an important monograph is [2]. It is worthy to indicate that lots of papers on the Kichhoff theory of rods have been published since 1900's because of the biological applications of the theory. Moreover, a monograph [3] was published in 2005.

However in a voluminous literature on rods, no any equation of rods is deduced from the three-dimensional theory of elasticity. On the contrary, it is indicated by Sanders [4] that Synge and Chien [5] developed a theory of shells, which is deduced from the 3D theory of elasticity.

In fact, the rod theory can not be regarded as the one-dimensional case of the classical theory of elasticity. The reason is that there are no rotations as the independent variables in the classical theory of elasticity. Erickson and Truesdell [6] and Antman [7,2] indicated that the rod theory has to be constructed on the theoretical framework of Cosserat elasticity.

In the present paper, the Cosserat elasticity is reduced to the one-dimensional form in the curvilinear coordinates. Two approximations are assumed. The first is that all the differential elements on a cross section have the same curvatures with those of the central line of rods at the center of the cross section. The second approximation is the absence of the interaction between the longitudinal fibers in rods. In this manner a set of new equations of rods is developed, which is formally same to the Kirchhoff equations.

At an arbitrary point P on the axis of curved rods, a system of coordinates $\left(P-x_1x_2x_3\right)$ is fixed, in which x_3-axis is identified with the axis of rods and x_1, x_2-axes coincide with two principal axes of the cross-section of rods, respectively. The x_3-axis is also seen as an arc-coordinate denoted by s.

In the Cosserat elasticity, equations of equilibrium of stresses and couple stresses are

$$\nabla_j t_{ji} + p_i = 0 \tag{1}$$

and

$$\nabla_j m_{ji} + e_{ijk} t_{jk} + q_i = 0 \tag{2}$$

where ∇_i is the covariant derivative with respective to x_i-axis. t_{ij} and m_{ij} denote stresses and couple stresses, respectively. p_i and q_i are body forces and body couples, respectively. e_{ijk} is a permutation symbol.

Consider a deferential element with edges parallel to x_1, x_2, x_3-axes, respectively, in curved rods. It is assumed that there exist only stresses t_{3i} and couple stresses m_{3i} on the element surface normal to x_3-axis, and the stresses and couple stresses on the other elements surface vanish:

$$t_{11} = t_{12} = t_{13} = t_{21} = t_{22} = t_{23} = 0, \quad m_{11} = m_{12} = m_{13} = m_{21} = m_{22} = m_{23} = 0 \qquad (3)$$

A substitution from (3) will simplify (1) and (2). Integrating the simplified equations over the cross section of rods leads to

$$\begin{aligned} &\frac{\partial F_1}{\partial s} + F_3\omega_2 - F_2\omega_3 + P_1 = 0 && \frac{\partial M_1}{\partial s} + \omega_2 M_3 - \omega_3 M_2 - F_2 + Q_1 = 0 \\ &\frac{\partial F_2}{\partial s} + F_1\omega_3 - F_3\omega_1 + P_2 = 0, && \frac{\partial M_2}{\partial s} + \omega_3 M_1 - \omega_1 M_3 + F_1 + Q_2 = 0 \\ &\frac{\partial F_3}{\partial s} - F_1\omega_2 + F_2\omega_1 + P_3 = 0 && \frac{\partial M_3}{\partial s} + \omega_1 M_2 - \omega_2 M_1 + Q_3 = 0 \end{aligned} \qquad (4)$$

where $\omega_i = \nabla_3 \phi_i$, ϕ_i is micro rotations, $F_i = \iint t_{3i} dA$ is the resultant forces acting on the cross section of rods with area A, $M_i = \iint (m_{3i} + \mu_i) dA$ is the resultant moments, $\mu_1 = t_{33}x_2$, $\mu_2 = -t_{33}x_1$ and $\mu_3 = -t_{31}x_2 + t_{32}x_1$.

It is found that the whole set of equations (4) is same to the Kirchhoff equations if the body force per unit length of rods P_i and body couple per unit length of rods Q_i are neglected.

By using (3) the resultant moments can be denoted in terms of curvatures in the form:

$$M_1 = A_1\omega_1, \quad M_2 = A_2\omega_2, \quad M_3 = A_3\omega_3 \qquad (5)$$

A substitution from them into the second row of (4) gives

$$\begin{aligned} &A_1 \frac{\partial \omega_1}{\partial s} + (A_3 - A_2)\omega_2\omega_3 - F_2 + Q_1 = 0 \\ &A_2 \frac{\partial \omega_2}{\partial s} + (A_1 - A_3)\omega_3\omega_1 + F_1 + Q_2 = 0 \\ &A_3 \frac{\partial \omega_3}{\partial s} + (A_2 - A_1)\omega_1\omega_2 + Q_3 = 0 \end{aligned} \qquad \text{the(6)}$$

Now consider inertial forces and inertial moments. Define first the angular velocity of the cross section as follows:

$$\Omega_1 = \frac{D\phi_1}{Dt}, \quad \Omega_2 = \frac{D\phi_2}{Dt}, \quad \Omega_3 = \frac{D\phi_3}{Dt} \qquad (7)$$

where D/Dt indicates that

$$\frac{D\phi_i}{Dt} = \frac{\partial \phi_i}{\partial t} + \phi_m \Gamma_{m3i}^{(t)} \qquad (8)$$

in which $\Gamma_{m3i}^{(t)}$ is Christoffel symbol.

By using $\boldsymbol{v} = \boldsymbol{v}(s,t) = (v_1, v_2, v_3)$ to represent the velocity of the point on the axis of rods, the inertial force is written as $\rho A \partial \boldsymbol{v}/\partial t$ and the inertial torque as $\partial(J \cdot \boldsymbol{\Omega})/\partial t$, where ρ is density and $J = diag(J_1, J_2, J_3)$ is moments of inertial.

By adding $\rho A\partial\mathbf{v}/\partial t$ and $\partial\left(J\cdot\mathbf{\Omega}\right)/\partial t$ on the right side of (4) and (6), respectively, and then expanding them according to (8), the final equations are obtained as

$$
\begin{aligned}
&\frac{\partial F_1}{\partial s}+F_3\omega_2-F_2\omega_3+P_1=\rho A\left(\frac{\partial v_1}{\partial t}+\Omega_2 v_3-\Omega_3 v_2\right)\\
&\frac{\partial F_2}{\partial s}+F_1\omega_3-F_3\omega_1+P_2=\rho A\left(\frac{\partial v_2}{\partial t}+\Omega_3 v_1-\Omega_1 v_3\right)\\
&\frac{\partial F_3}{\partial s}-F_1\omega_2+F_2\omega_1+P_3=\rho A\left(\frac{\partial v_3}{\partial t}+\Omega_1 v_2-\Omega_2 v_1\right)
\end{aligned}
\tag{9}
$$

and

$$
\begin{aligned}
&A_1\frac{\partial\omega_1}{\partial s}+\left(A_3-A_2\right)\omega_2\omega_3-F_2+Q_1=J_1\frac{\partial\Omega_1}{\partial t}-\left(J_2-J_3\right)\Omega_2\Omega_3\\
&A_2\frac{\partial\omega_2}{\partial s}+\left(A_1-A_3\right)\omega_3\omega_1+F_1+Q_2=J_2\frac{\partial\Omega_2}{\partial t}-\left(J_3-J_1\right)\Omega_3\Omega_1\\
&A_3\frac{\partial\omega_3}{\partial s}+\left(A_2-A_1\right)\omega_1\omega_2+Q_3=J_3\frac{\partial\Omega_3}{\partial t}-\left(J_1-J_2\right)\Omega_1\Omega_2
\end{aligned}
\tag{10}
$$

Two sets of equations (9) and (10) together are 6 equations for the 12 unkowns F_i, ω_i, v_i and Ω_i $\left(i=1,2,3\right)$. This demands 6 additional kinematic relations. Fortunately, the kinematic relations can be found in literatures. These equations are nonlinear. By using them the finite deformation of curved rods can be conveniently described.

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Professor Ranjan Banerjee

After receiving his Bachelor's and Master's Degree in Mechanical Engineering from the University of Calcutta and the Indian Institute of Technology, Kharagpur, respectively, Ranjan Banerjee joined the Structural Engineering Division of the Indian Space Research Organisation, Trivandrum in 1971 and worked there for four years, first as a Structural Engineer and then as a Senior Structural Engineer. He was involved in the dynamic analysis of multistage solid propellant rocket structures using the finite element method. He also carried out research on the response of rocket structures to acoustic loads. Later in the year 1975 he was awarded a Commonwealth Scholarship to study for a PhD degree at Cranfield University where he conducted research within the technical areas of structural dynamics and aeroelasticity. He received his PhD in 1978. An important spin-off from his PhD was the development of an aeroelastic package in Fortran, called CALFUN (CALculation of Flutter speed Using Normal modes) which was originally written for metallic aircraft, but later extended to composite aircraft. CALFUN has been extensively used as a teaching and research tool in aeroelastic studies. After completing his PhD, he joined the Structural Engineering Division of the University of Cardiff in 1979 and worked there for six years first as a Research Associate and then as a Senior Research Associate to investigate the free vibration and buckling characteristics of space structures using the dynamic stiffness method. During this period he worked in close collaboration with NASA, Langley Research Center, and he was principally involved in the development of the well-established computer program BUNVIS (BUckling or Natural VIbration of Space Frames) which was later used by NASA and other organizations to analyse spacecraft structures. He joined City University London in 1985 as a Lecturer in Aircraft Structures and he was promoted to Senior Lecturer and Reader in 1994 and 1998 respectively. In March 2003 he was promoted to a Personal Chair in Structural Dynamics. His main research interests include dynamic stiffness formulation, aeroelasticity, unsteady aerodynamics, composite structures, functionally graded materials, aircraft design, symbolic computation, free vibration and buckling analysis of structures and associated problems in elastodynamics. He has been responsible for supervising various research contracts as Principal Investigator, involving EPSRC, American Air Force Base, Embraer Aircraft Company, amongst others. To date he has published 102 journal papers and 90 conference papers from his research. He serves in the Editorial Boards of a number of international journals and established conferences and he has been a member of the EPSRC Peer Review College since its inception. He is a Fellow of both the Royal Aeronautical Society and the Institution of Structural Engineers in the UK and an Associate Fellow of the American Institute of Aeronautics and Astronautics. He teaches the subjects of mechanics, strength of materials, aircraft structures, composite materials, computational structural mechanics and aeroelasticity, and he has acted as external examiner in five British universities for their undergraduate and postgraduate programmes in aeronautical and aerospace engineering. He is a recipient of the Hind Rattan Award 2015.

Dipartimento Ingegneria Civile e Architettura

Università degli Studi di Catania, Italy

Salvatore Caddemi, Ph.D.

Prof. Salvatore Caddemi is native of Noto (Siracusa), Italy, on 29th of November 1960. He received his master degree in Civil Engineering in 1984, served as Officer in the Italian Navy until 1986 and obtained his Ph.D. in Structural Engineering in 1990, from the University of Palermo.

His research activity in the period 1988-1991 has been developed at the ”FRD/UCT Centre for Research in Computational and Applied Mechanics” of the University of Cape Town, South Africa, as “Visiting Researcher” and “Postdoctoral Research Fellow” contributing to theoretical advances in the integration of nonliner plastic constitutive laws and formulating iterative procedures for the relevant incremental analysis.

Prof. Caddemi was appointed Researcher of Mechanics of Materials in July 1991 at the Department of Structural and Geotechnical Engineering of the University of Palermo and was “Visiting Researcher” within the program HCM network of the European Community at the Department of Structural Engineering and Materials of the Technical University of Denmark in 1996.

In November 1998 he became associate professor of “Strenght of Materials” at the Institute of Structural Engineering of the Engineering Faculty of the University of Catania and since 1 october 2001 he is full professor at the Department of Civil and Architectural Engineering of the University of Catania where he is currently conducting his teaching and research activity.

His research interest has been oriented to deterministic analysis of elastic-plastic and no-tension material structures, stochastic dynamic structural analysis, structural and damage identification, static and dynamic analysis of structures with singularities, seismic vulnerability assessment of masonry structures.

Prof.Caddemi is currently involved in the use of generalised functions for the solution of direct and inverse problems of beam-like and frame structure in presence of strong discontinuites and singularities.

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Biosketch of Erasmo Carrera.

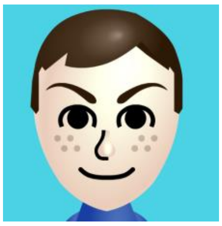
Erasmo Carrera graduated in Aeronautics in 1986 and in Space Engineering in 1988 at the Politecnico di Torino. He obtained a PhD in Aerospace Engineering in 1991. He became Assistant professor in 1992, Associate Professor (2000) and Full Professor (2010) at the Politecnico di Torino.

He has been visiting Professor at the Institute of Static and Dynamics of the University of Stuttgart , he spent 3 months at the department of Engineering Science and Mechanics of Virginia Tech in 1996; 3 months at Supmeca, Paris, in 2006; and 3 months at the Centre of Research Public H Tudor (Luxembourg) in the summer of 2009. In 2013 as HiCi Scientist Carrera has been member of Distinguished Professors Board at King Abdulaziz University, Jeddah, Saudi Arabia. During 2014 he had a joint appointment at School of Aerospace, Mechanical and Manufacturing Engineering at Royal Melbourne Institute of Technology of University of Melbourne, Australia.

Carrera has introduced the Reissner Mixed Variational Theorem, RMVT, as a natural extension of the Principle of Virtual Displacement to layered structure analysis. Moreover, he has introduced the Unified Formulation, or CUF (Carrera Unified Formulation), as a tool to establish a new framework in which to develop theories of beams, plates and shells for metallic and composite multilayered structures loaded by mechanical, thermal electrical and magnetic loadings. CUF is of a way to enhance to axiomatic and asymptotic approaches in the theory of structures.
Carrera has been author and coauthor of about 500 papers on the above topics, most of which have been published in first rate international journals, including three recent books published by J Wiley & Sons.

Dr Carrera has been responsible for various research contracts granted by public and private national (including regional ones) and international institutions, such as IVECO, the Italian Ministry of Education, the European Community, European Space Agency, Alenia Spazio Thales Alenia Space, Regione Piemonte, Embraer (Brasil). Among others he has been responsible for the structural design and analysis of full composite aircraft, named Sky-Y, by Alenia Aeronautica Torino, the first full-composite UAV made in Europe. He acts as Coordinator of the H2020 MSCA project 'FULLCOMP'.

Professor Carrera is founder and leader of the MUL2 group at the Politecnico di Torino. The MUL2 group is considered one of the most active research team at Politecnico; it has acquired a significant international reputation in the field of multilayered structures subjected to multifield loadings, see also www.mul2.com.
Professor Carrera is Highly Cited Researchers (Top 100 Scientist) by Thompson Reuters in the two Sections: Engineering and Materials.



Sergio De Rosa
Professor
Department of Industrial Engineering – Aerospace Section
University of Naples “Federico II”, Italy

Short Bio
From the beginning of my research activity, I was involved in the engineering fields concerning the structural dynamics, the vibroacoustics and the fluid-structure interaction in large sense.

The thesis work concluded the five-year Italian degree in aerospace engineering in 1998. It was centred on the assembling of a predictive vibroacoustic model for the ATR42 fuselage. Large part of that time was usefully spent on the pages of vibration of plates and shells. In the same year, I became researcher at the Italian Aerospace Research Center, CIRA, where continued my studies in the vibroacoustic fields.

In 1992, I arrived as researcher at University of Naples “Federico II” at the Department now named Industrial Engineering - Aerospace Section: my entire academic career was spent in the Aerospace Structures sector under the guide - often controversial - of my friends Leonardo (Lecce) and Francesco (Marulo). In parallel with the research activity, I was also guided to the teaching stuff, having from 2001 the full responsibility of courses. Since 1990 I was involved in the main research programmes funded by the European Union in the aerospace sector.

The targets of the applied and base research activities were to study numerical and experimental procedures and/or tools to be used as predictive methods, all inside the paradigm of increasing complexity. My actual research topics are:

- Models for the structural dynamics and interior acoustics
  - definition of ASMA, Asymptotical Scaled Modal Analysis;
  - definition of SAMSARA, Similitudes and Asymptotical Modelling for Structural Acoustics Researches and Applications.
- Stochastic response of structural and fluid-structural systems under random and convective excitations.
- Convective effect on the acoustic radiated power by structural components.
- Influence of the uncertainties on the dynamic system response.

I am still convinced that only a full interaction among human beings can promote the knowledge at high levels. The research and teaching mechanics can’t be a passive process but requires that all the involved persons play something, hopefully with a smile ☺.

This is the second time for me to be at ISCVS, and I will be very glad to meet agaib some of the persons who guided my adventure. Nevertheless, I am (very) lazy researcher but I know to be a loquacious friend, being passionate of photography, blogging and micro-blogging, music, modern literature, comics.

I may say to know many things: all equally bad, obviously.

BIOSKETCH

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1. **H Ding**, LQ Chen, Stability of axially accelerating viscoelastic beams multi-scale analysis with numerical confirmations, *European Journal of Mechanics A/Solids*. **2008**, 27: 1108–1120;
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2. National Natural Science Foundation of China (Project No. 11372171), “Nonlinear vibration of pulley-belt drive systems coupled with a one-way clutch”, January 1, 2014 to December 31, 2017, Principal Investigator;
3. National Natural Science Foundation of China (Project No. 10902064), “Transverse Vibrations of Axially Moving Viscoelastic Beams in the supercritical regime: Modeling, Analysis, and Simulation”, January 1, 2010 to December 31, 2012, Principal Investigator.

HONORS

1. Nomination Award of National Excellent Doctorate Dissertation of China, 2010;
2. Scholar of Shanghai Rising-Star Program, 2011;
3. Excellent young scientist of NSFC, 2014.

BIOGRAPHICAL INFORMATION

Earl H. Dowell
William Holland Hall Professor
Pratt School of Engineering
Duke University

Dr. Dowell is an elected member of the National Academy of Engineering, an Honorary Fellow the American Institute of Aeronautics and Astronautics (AIAA) and a Fellow of the American Academy of Mechanics and the American Society of Mechanical Engineers. He has also served as Vice President for Publications and member of the Executive Committee of the Board of Directors of the AIAA; as a member of the United States Air Force Scientific Advisory Board; the Air Force Studies Board, the Aerospace Science and Engineering Board and the Board on Army Science and Technology of the National Academies; the AGARD (NATO) advisory panel for aerospace engineering, as President of the American Academy of Mechanics, as Chair of the US National Committee on Theoretical and Applied Mechanics and as Chairman of the National Council of Deans of Engineering. From the AIAA he has received the Structure, Structural Dynamics and Materials Award, the Von Karman Lectureship and the Crichlow Trust Prize; from the ASME he has received the Spirit of St. Louis Medal, the Den Hartog Award and Lyapunov Medal; and he has also received the Guggenheim Medal which is awarded jointly by the AIAA, ASME, AHS and SAE.

Currently he serves on boards of visitors of Carnegie Mellon University, Princeton University, the University of Illinois and the University of Rochester. He is a consultant to government, industry and universities in science and technology policy and engineering education as well as on the topics of his research.

Dr. Dowell research ranges over the topics of aeroelasticity, nonsteady aerodynamics, nonlinear dynamics and structures. In addition to being author of over three hundred research articles, Dr. Dowell is the author or co-author of four books, "Aeroelasticity of Plates and Shells", "A Modern Course in Aeroelasticity", "Studies in Nonlinear Aeroelasticity" and “Dynamics of Very High Dimensional Systems”. His teaching spans the disciplines of acoustics, aerodynamics, dynamics and structures.

Dr. Dowell received his B.S. degree from the University of Illinois and his S.M. and Sc.D. degrees from the Massachusetts Institute of Technology. Before coming to Duke as Dean of the School of Engineering, serving from 1983-1999, he taught at M.I.T. and Princeton. He has also worked with the Boeing Company.

Lorenzo Dozio

Department of Aerospace Science and Technology, Politecnico di Milano, Italy

Lorenzo Dozio was born near Milan, Italy, on 1972. He received a M.S. degree in Aerospace Engineering in 1998 and a Ph.D. in Aerospace Engineering in 2002, both at the Politecnico di Milano. After two years as a post-doc, he won a position as Assistant Professor at the same University in 2004. Last year, he got the Italian National Scientific Qualification as Associate Professor in Aeronautics, Aerospace and Naval Engineering.

Since 2002 he has been involved in teaching activities concerning servosystems for aerospace applications, introduction to engineering experimentation and dynamics and control of aerospace structures.

His main research interests are vibration of structures, composite and smart materials, active and shunt piezoelectric control, coupled structural-acoustic and real-time control systems. He has been involved in many research projects in collaboration with industries on active noise reduction inside helicopter cabins, active control of instabilities in combustion chambers and design and implementation of real-time operating systems. He is currently working on refined computational and analytical models for bending, vibration and buckling analysis of multilayered plates and shells.

He has authored more than 20 papers in international journals and over 50 conference papers. He has advised about 30 graduate students at Politecnico of Milano. He served as a reviewer for, among others, Journal of Sound and Vibration, Journal of Vibration and Acoustics, Composite Structures and International Journal of Mechanical Sciences.

He is married to Letizia, and they have four children, two sons Paolo (13) and Tommaso (11), and two twin daughters Anna and Matilde (7). In his spare time, he loves playing acoustic and electric guitar.

Mark S. Ewing
University of Kansas

I grew up interested in science and mathematics, largely due to my fascination with the U.S. spacecraft I used to watch launch from Cape Canaveral—when I lived in nearby Orlando, Florida. I received my BS in Engineering Mechanics from the U.S. Air Force Academy, then began a 20-year career in the Air Force. I served for four years in turbine engine stress and durability analysis where I was an "early" user of finite element analysis for hot, rotating turbomachinery. I then served a two-year assignment in turbine engine maintenance and support, which was less technical, but eye-opening. During these early years—in my spare time—I earned an MS in Mechanical Engineering from Ohio State University.

With an MS in hand, I returned to the Air Force Academy to serve on the faculty as an Assistant Professor. After two years, I returned to Ohio State to complete a PhD. As a student of Art Leissa's, I focused on the combined bending, torsion and axial vibrations of "stubby" beams, thereby establishing my interest in the vibrations of continuous systems.

After returning to and teaching at the Academy for six years, I was assigned to the Air Force Flight Dynamics Lab, where I worked on two interesting projects. The first was the development of a structural design algorithm capable of, among other things, "maximizing" the separation of two natural frequencies. The utility of this endeavor was to allow the design of aircraft wings for which the bending and torsional natural frequencies are sufficiently separated (in frequency) to avoid flutter. The other interesting project was the analysis of the effect of convected aerodynamic loads on a missile.

I am now on the Aerospace Engineering faculty at the University of Kansas. My current research interests are in structural acoustics, which is a topic of increasing interest to aircraft manufacturers. In recent years, I have focused on the best way to characterize and estimate structural damping for built-up structures. All the test articles I've used to validate my work through experimentation are simple structural elements, namely beams and plates.

I have a great love of the outdoors, and of the mountains in particular. When Art Leissa asked me to help organize the first International Symposium on Vibrations of Continuous Systems—held in 1997—and he told me he wanted to meet in the mountains, I really got excited. I look forward to the 10th Symposium back where we started in Colorado as a time to visit with long-time friends and colleagues. I may not be as quick up the mountains as before, so I look forward to the physical challenges as well.

Peter Hagedorn

Peter Hagedorn was born in Berlin, Germany. He grew up in Brazil, where he graduated (Engineer's degree) in mechanical engineering in 1964 at EPUSP and in 1966 earned his doctoral degree at the same University. He then worked as a research assistant and later as 'dozent' (similar to lecturer) at the University of Karlsruhe, Germany. In 1971 he got his 'habilitation' (similar to Dr. Sc.) at Karlsruhe. From 1973 to 1974 he was a visiting Research Fellow at the Department of Aeronautics and Astronautics, Stanford University. Since October 1974 he is full professor of mechanics at the Technische Universität Darmstadt and head of the Dynamics and Vibrations group. He also has served as visiting professor at Rio de Janeiro (Brazil), Berkeley, Paris, Irbid (Jordan) and Christchurch (New Zealand), where he also holds an Adjunct Professorship at UCC. He has served as Head of Department and Vice-President to his home University in Darmstadt and he is serving in a number of professional and editorial committees. He is author of over 200 papers and several books on a variety of topics in the general field of dynamics and vibrations and analytical mechanics. He is officially retired since 2009 but still quite active and heads the Dynamics and Vibrations Group, presently affiliated to the chair of professor Michael Schäfer, at the graduate school of computational engineering of TU Darmstadt.

BIOGRAPHY

Jared D. Hobeck received a B.S. degree in general engineering from Montana Tech at the University of Montana in 2008, an M. Eng. degree in mechanical engineering from Virginia Polytechnic Institute and State University in 2012, and a Ph.D. degree in aerospace engineering from the University of Michigan in 2014 where he is currently a Postdoctoral Research Fellow.

From 2009 to 2011 he was a Graduate Research Assistant in the Center for Intelligent Materials Systems and Structures and in the Center for Energy Harvesting Systems. Since 2007 he has performed research in multiple academic, private, and government labs. His research interests include linear and nonlinear structural dynamics, energy harvesting technologies, smart materials, flow-induced vibration, and composite structures.

Dr. Hobeck's honors and awards include a being an invited speaker at the United States Naval Research Laboratory in 2014, he received the Best Paper Award and was Best Hardware finalist at the ASME SMASIS Conference in 2011, he is a member of Tau Beta Pi Engineering Honor Society, and received the Outstanding Student of the Year Award for the general engineering department at Montana Tech in 2008.

Sinniah Ilanko
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Ilanko was born in the north of Sri Lanka (Jaffna), and according to the common Tamil practice, he does not have/use a family name. Ilanko is his given name and Sinniah is his late father's given name. He is a Professor and currently the Head of School of Engineering at the University of Waikato (NZ). Since January 2009 he is serving as the Subject Editor for Journal of Sound and Vibration, for analytical methods for linear vibration.

Ilanko graduated from the University of Manchester (U.K) with a BSc in civil engineering and also obtained an MSc from the same university under the supervision of late Dr S.C. Tillman. His move to England at an early age was due to the support and encouragement from his late brother Senthinathan. He completed his doctoral studies at the University of Western Ontario under the supervision of Professor S.M. Dickinson and then joined the University of Canterbury where he worked in various positions as a Lecturer, Senior Lecturer and then Associate Professor between 1986 and 2006. He joined the University of Waikato in 2006 and in 2012 he became a full professor.

Ilanko's research areas include vibration and stability of continuous systems, numerical modelling and adaptive mechanisms and he has published articles in these areas in the following journals: Journal of Sound and Vibration, Proceedings of the Royal Society A, International Journal for Numerical Methods in Engineering, Communications in Numerical Methods in Engineering, Journal of Applied Mechanics, Journal of Fluids and Structures and Computers and Structures. A book he jointly authored with Dr Luis Monterrubio with assistance from Dr Yusuke Mochida entitled "The Rayleigh-Ritz Method for Structural Analysis" was published by ISTE Wiley in 2014.

Ilanko is also interested in computer-aided learning and has developed and used several interactive lectures and tutorials for teaching Mechanics of Materials and Vibration, as well as computer tutorials and games for learning/teaching Tamil language. He maintains a "vibration resources homepage" (see the second URL above), which at present contains some interactive simulation programs for calculating natural frequencies and modes of some structural elements.

Ilanko is married to Krshnanandi and they have two daughters, Kavitha and Tehnuka. Ilanko's birth family is scattered across the globe (Australia, Canada, New Zealand, the U.K. and the U.S.A.) because of the civil war in Sri Lanka.

David Kennedy
Professor of Structural Engineering
Cardiff School of Engineering, Cardiff University, United Kingdom

David Kennedy obtained a First Class Honours degree at the University of Cambridge in 1978 and a PhD in the area of efficient transcendental eigenvalue computation from the University of Wales, Cardiff in 1994.

From 1978 to 1983 he was employed as an Analyst/Programmer for the computer services company Scicon Ltd, where he worked on the development of the Mathematical Programming software SCICONIC/VM. In 1981 he was awarded a 2-year BP Venture Research Fellowship in Non-linear Optimization, supervised by the late Professor Martin Beale.

In 1983 he was appointed as a Research Associate in the University of Wales Institute of Science and Technology, which was merged into Cardiff University in 1988. Working under the supervision of Professor Fred Williams and funded under a collaborative agreement with NASA, he co-ordinated the development of the space frame analysis software BUNVIS-RG which was released by NASA to US users in 1986/87. Further collaboration with NASA and British Aerospace (now BAE Systems) led to the development and successive releases, starting in 1990/91, of VICONOPT, a buckling and vibration analysis and optimum design program for prismatic plate assemblies. Both of these programs use analysis methods based on the Wittrick-Williams algorithm.

He was appointed to a Lectureship in the Cardiff School of Engineering in 1991, promoted to Senior Lecturer in 2000, Reader in 2005 and Professor in 2009. He has continued to manage the collaborative development of VICONOPT, successfully co-supervising 16 PhD students and holding Research Council grants on parallel computing, aerospace panel optimization, local postbuckling and mode finding. He has visited NASA Langley Research Center several times, and in 2007 he undertook a 6-month secondment to Airbus UK, funded by a Royal Society Industry Fellowship. In 2010 he was appointed as a Deputy Director of the Cardiff School of Engineering with responsibility for staff matters.

Through the Cardiff Advanced Chinese Engineering Centre, Professor Kennedy has participated for over 20 years in collaborative research projects with leading Chinese universities, including Tsinghua University, Dalian University of Technology and Shanghai Jiao Tong University.

Professor Kennedy is the author of 169 publications of which approximately 50% are in refereed journals of international standing.

He lives with his wife Helen and enjoys choral singing, hill walking and running (though at a slower pace than he once did).

Arthur W. Leissa

Professor Emeritus, Ohio State University

After earning two degrees in mechanical engineering, with a strong interest in machine design, I decided to seek better understanding of stress and deformation of bodies, so I got my Ph.D. in engineering mechanics (from Ohio State University in 1958). My dissertation research was in the theory of elasticity. I then stayed on as a faculty member.

Working part-time for two major aircraft companies (Boeing and North American Aviation) made me very interested in vibrations. In 1965 I approached NASA to support me with research funds to collect the literature of the world in plate and shell vibrations, and summarize it in two monographs. They did, and the two books were published in 1969 and 1973. They were out of print for a long time. But in 1993 they were reprinted by The Acoustical Society of America and are currently available from them.

After gaining considerable knowledge in writing the two books, I continued to do extensive research on vibrations of continuous systems, including laminated composites turbomachinery blades, and three-dimensional problems. Approximately 150 published papers, and most of the 40 doctoral dissertations I supervised, were in this field

I always intended to update the "Vibration of Plates" monograph. Indeed, more than 20 years ago I had a graduate student collect the more recent literature. This consisted of 1500 additional references dealing with free vibrations of plates. But I never could find the time needed to undertake the writing.

In June of 2001 I formally retired from Ohio State University after having been on its faculty for 45 years. During that time I taught a graduate-level course "Vibrations of Continuous Systems " each year. In July, 2002 Trudi and I moved to Fort Collins, Colorado, approximately 60 miles north of Denver, and close to the mountains. In 2011 I did manage to complete the textbook "Vibrations of Continuous Systems", published by McGraw-Hill.

My serious interest in the mountains began as a boy, reading books about Mallory and Irvine on Everest, and others. In 1961 when I could first afford it (with a family) I began climbing mountains, which I pursued strongly for decades. Among my climbs were all the highest mountains in Colorado, one-half of the highest Swiss Alps, and Mount McKinley (20,000 ft) in Alaska. Now being 83, I can no longer climb them, but I still enjoy greatly being in the mountains---hiking, skiing and snowshoeing.. They restore one's vitality and one's peace.

In 1995 Mark Ewing, who was in Colorado then, agreed to help me organize the first International Symposium on Vibrations of Continuous Systems, held in 1997 in Estes Park, Colorado. It was well received, and so it has continued every two years in marvelous mountain locales worldwide. I look forward to taking part again, this time in a return to Estes Park, Colorado. My mind is definitely slipping, and my body and legs can no longer endure significant hikes, but I shall enjoy participating in what ways I can.

Shinichi Maruyama
Gunma University

Shinichi Maruyama is an associate professor of the Department of Mechanical System Engineering in Graduate School of Engineering, Gunma University, Japan.

He was born in Takamatsu and had been lived in Chiba, suburb area of Tokyo, until he graduated university. He obtained Master of Engineering and Doctor of Engineering in 1999 and 2002, both from Keio University. Since 2002, he has been taking an academic position in Gunma University and working with Professor Ken-ichi Nagai.

His research interests include nonlinear and chaotic vibrations of mechanical systems, and analyses and experiments on dynamics of thin elastic structures.

He is a member of the Japan Society of Mechanical Engineers. Since 2010, He has been the chair of the Technical Section on Basic Theory of Vibration in the Division of Dynamics, Measurement and Control in JSME.

YUSUKE MOCHIDA

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I currently hold a Postdoctoral Research/Teaching Fellowship at the University of Waikato in New Zealand. The overall aim of my current research is to develop a general analytical procedure for vibration analysis of complex structures. The method under consideration includes the use of the concept of negative structures to represent voids.

I was born and grew up in Japan. After I graduated with a B.E. in Mechanical Engineering from the Tokyo Metropolitan University (Japan) I worked for a while in Japan and went to New Zealand as a working holiday maker to travel around and work. Actually I was away from the engineering field for several years. This made me miss engineering and so after learning English, I enrolled in a Postgraduate Diploma programme at the University of Canterbury (New Zealand). During my postgraduate study I became interested in vibration and decided to continue towards an M.E. under the supervision of Professor Ilanko, who had at this time relocated to the University of Waikato. I completed my M.E. and then continued working towards a Ph.D at the same university. Since commencing my M.E. studies I have developed several codes based on the Superposition Method, the Rayleigh-Ritz Method and the Finite Difference Method to solve free vibration problems of plates and shells using MATLAB. In addition to my research experience, I lectured in Dynamics and Mechanisms and tutored in Vibration, Mechanics and Finite Element Analysis classes.

Through my career, I hope I can contribute to the development of research relationships between New Zealand and other countries, especially Japan, and the advancement of research in New Zealand.

Personally, I am also interested in snowboarding, golf, playing drums, Shorinji Kempo (Japanese martial arts), foreign exchange, personal development and cooking.

Luis Monterrubio, Ph.D. Mechanical Engineering

Assistant Professor of Mechanical Engineering

Luis Monterrubio joined the Robert Morris University Engineering Department as an Assistant Professor in the Fall of 2013. He earned B.Eng. from the Universidad Nacional Autónoma de México, a M.A.Sc. from the University of Victoria, Canada and his Ph.D. from the University of Waikato, New Zealand. All degrees are in Mechanical Engineering and both M.A.Sc. and Ph.D. studies are related with vibrations. After his Ph.D. he worked at the University of California, San Diego as postdoctoral fellow in the area of bioacoustics.

He teaches dynamics, machine design, numerical methods and finite element method. His research interests are in vibration, numerical methods, finite element methods, continuum mechanics, acoustics and engineering education

He has work for the automotive industry in drafting, manufacturing, testing (internal combustion engines –vibration, fatigue, thermo-shock, tensile tests, power, torque and exhaust emissions, etc.), simulations (finite element method) and as a project manager (planning and installation of new testing facilities).

He is a member of the American Society of Mechanical Engineers and the American Society for Engineering Education.

Ken-ichi Nagai
Gunma University, Japan

Ken is a professor emeritus in Gunma University since 2012.

He graduated from the national college of technology in Fukushima in 1967. In the student life, he received academic interest from the book "Mechanics" written by Den Hartog. He wanted to devote himself in areas of research and education. He received his B. Eng. in 1970 from Ibaraki University. He obtained Mr. Eng. and Dr. Eng. in 1972 and 1976 from Tohoku University, respectively. Main research subject was nonlinear vibrations of plates and dynamic stability of plates and cylindrical shells, under his supervisor Professor Noboru Yamaki.

Since 1976, he got an academic position in Gunma University. From 1990 to 1991, he visited to the Cornell University in U.S.A. as a research fellow. During the stay in U.S.A., Professor Leissa in the Ohio State University gave him nice advises in research area. Then, he also visited to Professor Hagedorn in the Technische Hochshule Darmstadt in Germany. Furthermore, he visited Polish Academy of Sciences in Poland.

He is a Fellow of the Japan Society of Mechanical Engineers. He has been a academic consultant to the local government.

He has an interest in the filed of nonlinear dynamics and chaotic vibrations of thin elastic structures such as beam, arch, plate and shell. Recently, he published the books of "Dynamical system Measurements", "Dynamic system Analysis -Energy Approaches from Structural Vibration to Chaos-" and “Dynamics of Nonlinear Systems –Introduction to Analysis of Nonlinear Phenomena-”.

His is now interesting in a two hour trecking up to the mountain near his home. He feels spiritual happiness by walking in fields of nature and by facing to chaotic phenomena generated from dynamical system.

Yoshi (Yoshihiro Narita)

Hokkaido University, Sapporo, Japan

I am a professor of Mechanical Engineering at Hokkaido University, Sapporo. Hokkaido is the name of island northern most among four large islands in Japan. Eleven years ago, I moved to Hokkaido University (HU), and I like teaching young students and working with graduate students. I enjoy freedom of doing research and visiting international conferences and foreign universities. Associate professors, Dr.Honda (solid mechanics, optimization) and Assistant Professor Dr.Lee (ergonomics, affective engineering), are working hard to support our laboratory with fifteen Japanese and foreign graduate students.

I started my research on vibration of continuous systems when I was a PhD student under adviser Prof.Irie of HU in 1976, and had a chance to study one year in 1978-1979 under Prof.Leissa at the Ohio State University. The research outcomes under both advisers were summarized into my PhD dissertation in 1980 with the title "Free Vibration of Elastic Plates with Various Shapes and Boundary Conditions". Even after 35 years, it is downloaded more than 17000 internationally (ex. 7300 in China, 930 times in USA, 400 times in India and so on) from HUSCAP website: http://eprints.lib.hokudai.ac.jp/dspace/handle/2115/32630.

I used to play tennis and ski. Now, traveling abroad may be my hobby, which is inseparable from my profession. Last April and May, my wife and I travelled to Buenos Aires, Falls of Iguasu both in Argentine and Brazil, and also Machu Picchu in Peru. Furthermore, from April, I became the director of Helsinki Europe Office of Hokkaido University, and almost every month I commute to the office in downtown Helsinki!

I am very delighted with visiting Estes Park again after eighteen years. I could join in all the ISVCS's, and these symposiums are full of good memories. In the present ISVCS-10, I look forward to meeting old and new friends in the research community of vibration mechanics.

Let's enjoy!

Biography
of Francesco Pellicano

Francesco Pellicano was born in Rome, Italy on 1966. He received a M.S. degree in Aeronautical Engineering in 1992 and Ph.D. in Theoretical and Applied Mechanics in 1996, both at the University of Rome “La Sapienza”, Dept. of Mechanics and Aeronautics.

He was Researcher at the University of Modena and Reggio Emilia, Faculty of Engineering, Dept. of Mechanical and Civil Engineering, 1996-2003.

He is currently Associate Professor at the same University since January 2004 and vice-Head of the Centre Intermech MoRe.

He was involved in investigations concerning: nonlinear vibrations of structures; vibration control; axially moving systems; nonlinear vibration of shells with fluid structure interaction, vibration of carbon nanotubes; gears modeling; non-smooth dynamics; Chaos; Nonlinear Time Series Analysis; Forecasting Methods in Oceanography.

He cooperated with Prof. Vestroni, Prof. Sestieri and Prof. Mastroddi of the University of Rome “La Sapienza” and with: Prof. Païdoussis (Mc Gill Univ. Canada); Prof. Vakakis (Univ. of Illinois at Urbana Champaign); Prof. Amabili (Univ. of Parma, Italy), Prof. L. Manevitch (Semenov Inst. Moscow, Russia); Prof. Ilanko (Waikato Univ. New Zealand).

The teaching activity regards: Vibrations of Discrete and Continuous Systems; Signal Processing; Machine Theory and Machinery.

He is coordinator of two FP7 EU projects: INDGEAR (condition monitoring) and HPGA Fortissimo (applications of high performance computing); he was coordinator of several international and national projects.

His research activity regards also industrial problems, he cooperated for research and consultancies with several companies about: vibration control; experimental vibrations; simulation of mechanical systems.

He is Associate Editor of the journals: *Mathematical Problems in Engineering, Hindawi; Chaos, Solitons and Fractals, Elsevier; moreover,* he takes part *to* the international advisory editorial board of the journal: *Communications in Nonlinear Science and Numerical Simulation, Elsevier.*

He published 2 Books, about 50 Journal papers and more than 100 conference papers

Bio – data

I was born on July 05, 1948 in the village Rahimpur of the Munger district, Bihar, India. I grew up in Munger and completed elementary and secondary educations from the Munger Zila School. I was directly admitted to the Bihar Institute of Technology in Sindri from where graduated in 1968 with B.Sc. Eng. (first class with distinction in Mechanical Engineering). Then I came to Canada and joined the school of graduate studies at the University of Ottawa in September 1969. I began my research work with the derivation of the constitutive equations from the first principles to study the free axisymmetric vibration of sandwich spherical shell structures under the noble supervision of Professor S. Mirza and subsequently received M. A. Sc. and Ph. D. degrees. These equations were developed in the spherical coordinate system and had solutions in Legendre functions of complex order for which I had to develop many new programs. During this study I also used energy methods to deduce the equations of motion for the free vibration of isotropic and sandwich plates and shells.

After graduation, I worked as a defence scientist at the Defence Research Establishment Suffield (DRES) near Medicine Hat Alberta from January 1978 to April 1981. Then, I accepted a design-engineer position in the Civil Design Department of Ontario Hydro in Toronto and worked there until December 1984, when I came to the Western University to teach machine component design and the finite element methods. Professor Stuart Dickinson was the chair of the Mechanical Engineering at that time and he is the one who hired me. This year I shall be completing 31 years of service to the university. During these years I taught other courses such as graphics and engineering drawings, dynamics, kinematics and dynamics of machines, the modern control systems, theory of plates and shells, continuum mechanics, computational methods in engineering to name a few. I worked with some remarkable students in the field of computational solid mechanics dealing with the linear and nonlinear vibrations of plates and shells.

About the family: I am married to Bimla since the March of 1968. We have two grown up children, the son Bidhi and daughter Shikha who are graduates of the Western University. Bidhi is married to Swati and they have a son named Akshaj. Bimla and I attended seven of the last nine ISVCS symposia. This is the eighth one. We like to travel, camp, enjoy walking in parks and on the beaches (whenever possible), etc. I have given notice of retirement to the Western University effective July 01, 2016 after which wish to live to the fullest as long as there is wellness.

Anand V. Singh, Ph.D., P. Eng.
Professor
Department of Mechanical and Materials Engineering
The Western University, London, Ontario, Canada, N6A 5B9

Andrew Watson

Lecturer of Aerospace Structures

Department of Aeronautical and Automotive Engineering

Loughborough University, United Kingdom

Andrew obtained his undergraduate and higher degrees from Cardiff University. His PhD looked at the stability analysis and optimisation of light weight structures. After two post-doctoral appointments at Cardiff Andrew joined Loughborough University as a member of academic staff in 2004.

His research includes buckling and postbuckling of aerospace panels; vibration of beams and quantum graphs. Buckling, vibration and quantum graph problems can be approached by using the Wittrick-Williams algorithm. Andrew is currently developing a set of tools to provide the user with eigenvalue and eigenvector solutions to any shaped graph with a differential operator.

Other research includes experimental and numerical testing of damage in composite panels and numerical simulation of geometric nonlinear beams subject to bending and compression. More recently Andrew has been looking at fossil fuels and other finite resources. He has taken a very keen interest in energy demand reduction and has delivered talks on the subject at various academic institutions and schools.

In his spare time he likes to keep up with current affairs and enjoys walking, cycling and open water swimming.

CURRICULUM VITAE

Ruojing ZHANG
Professor (retired in 2011)
Institute of Applied Mechanics
Tongji University
Shanghai 200092, CHINA

Education

1964 - 1970	PEKING university, Beijing, CHINA **B.S.** in math-mechanics
1979 - 1982	TSINGHUA university, Beijing, CHINA **M.S.** in solid mechanics
1984 - 1988	TSINGHUA university, Beijing, CHINA **PH.D** in solid mechanics

Career -Related Activities:

1. **Engineer**, Commissariat a l'Energie Atomique, Center d'Etudes Nucleaires, FRANCE, 1994-1995
2. **Visiting professor**, Nagoya University, JAPAN, 1996-1997 (3 months)
3. **Research Fellow**, the Hong Kong Polytechnic University, 1998 (3 months), 1999 (2 months)
4. **Visitor,** University of Science and Technology of Hong Kong 2001 (2 months)
5. **Visiting professor**, University of Sydney, AUSTRALIA, 2002 (3 months)

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